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One-dimensional bi-generalized diffusion processes

Dedicated to Professor Nobuyuki Ikeda on his 60th birthday

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1. Introduction.

The theory of one-dimensional diffusion processes (ODDPs for brief) was extensively developed in 1950's by many mathematicians headed by W. Feller, K. Itô, E. B. Dynkin, H. P. McKean and so on (see the references of [**9**] for the literatures). An ODDP is a strong Markov process with continuous sample paths, and it is determined by the strictly increasing continuous scale function s and the positive speed measure dm on an interval in the real line. The positivity of dm was soon relaxed to nonnegativity, and appeared the notion of generalized diffusion processes (GDPs) or gap processes. A GDP is a strong Markov process with right continuous sample paths, which may jump only to the nearest neighbours in the support of dm, and it is determined by a strictly increasing continuous scale function s and a nonnegative speed measure dm. The set of ODDPs or GDPs forms an effective and beautiful class from both probabilistic and analytic points of view. However, in the recent development of their application, there appeared a one-dimensional Markov process corresponding to the scale function with jumps and the Lebesgue speed measure (see [**7**] and [**12**]). In our introductory lecture [**13**], we tried to define the class of those processes by means of the expression $s^{-1}\circ B(\mathfrak{f}^{-1}(t))$, where B is a Brownian motion and $\mathfrak{f}$ is a random time change function. But it remained to reveal the behavior of the process on the flats of s, when they exist.

In this paper, we first define and construct the one-dimensional Markov process corresponding to a non-decreasing scale function s and a nonnegative speed measure dm, which we call a bi-generalized diffusion process (BGDP). The obtained process neither is strong Markov nor has right continuous sample paths in general anymore. Actually, there are 'chaotic' ponds, where the sample paths are absolutely jumbled, but after identifying each such pond as one point, the sample paths are quite tame; they are right continuous and jump only to the nearest neighbours in the support of dm. This situation is realized by our auxiliary GDP Y given in the following sections, which, I believe, is the same as Ray-Knight process; it is right continuous strong Markov process and the state