J. Math. Soc. Japan
Vol. 41, No. 1, 1989

On small data scattering with cubic convolution nonlinearity

Dedicated to Professor Takeyuki Hida on his sixtieth birthday

By Kiyoshi MOCHIZUKI

(Received Sept. 7, 1987)

1. Introduction.

We shall consider the Schrödinger equation

$$\frac{1}{i}\partial_t w=\Delta w+f(w), \tag{1.1}$$

the Klein-Gordon equation

$$\partial_t^2 w=\Delta w-w+f(w), \tag{1.2}$$

and the wave equation

$$\partial_t^2 w=\Delta w+f(w) \tag{1.3}$$

for $(x, t)\in \boldsymbol{R}^n\times\boldsymbol{R}$, where $i=\sqrt{-1}$, $\partial_t=\partial/\partial t$, $\Delta=\sum_{j=1}^n\partial_j^2$ $(\partial_j=\partial/\partial x_j)$ and $f(u)$ represents the cubic convolution nonlinearity:

$$f(w)=(V*|w|^2)w=\left(\int_{R^n}V(x-y)|w(y)|^2dy\right)w(x). \tag{1.4}$$

The steady state equations corresponding to (1.1), (1.2) and (1.3) have the same form and are given by

$$-\Delta v-f(v)=\mu v \qquad (\mu\in\boldsymbol{R}). \tag{1.5}$$

This equation has been studied e.g., in Gross [**6**], Lions [**10**] and Menzala [**12**]. In case $V=|x|^{-1}$, (1.5) is known as the Hartree equation for the helium atom. The time dependent equation (1.1) has been studied by Glassey [**5**], Ginibre-Velo [**4**], Dias-Figueira [**3**], Hayashi-Tsutsumi [**7**] and Hayashi-Ozawa [**8**], and equations (1.2) and (1.3) have been studied by Menzala-Strauss [**13**]. The positivity $V(x)\geqq 0$ and the symmetry $V(-x)=V(x)$ are required there. Then the well-posedness of the Cauchy problem and the asymptotic behaviors of solutions

This research was partially supported by Grant-in-Aid for Scientific Research (No. 62302004), Ministry of Education, Science and Culture.