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Integral arithmetically Buchsbaum curves in $\mathbf{P}^3$

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Introduction.

When a curve X (not assumed to be smooth nor reduced) in $\mathbf{P}^3$ has the property that its deficiency module $\bigoplus_n H^1(\mathcal{J}_X(n))$ is annihilated by the homogeneous coordinates x_1, x_2, x_3, x_4 of $\mathbf{P}^3$, it is called an arithmetically Buchsbaum curve. In [**1**], we defined a numerical invariant "basic sequence" of a curve in $\mathbf{P}^3$ (see [**1**; Definition 1.4]) and classified arithmetically Buchsbaum curves with nontrivial deficiency modules in terms of their basic sequences. But there, an important problem was left unconsidered; to find a necessary and sufficient condition for the existence of integral arithmetically Buchsbaum curves with a given basic sequence. The aim of this paper is to give a complete answer to this problem in the case where the base field has characteristic zero. The existence theorems for some special cases, e.g. [**1**; Theorem 4.4], [**2**; Corollary 2.6], [**3**; Proposition 4.7] and [**4**; pp. 125-126], are now corollaries to our general theorem.

NOTATION AND CONVENTION. The base field k is algebraically closed. We do not assume that char$(k)=0$ except in the main theorem. The word "curve" means an equidimensional complete scheme over k of dimension one without any embedded points. Given a matrix Φ, $\Phi\binom{i}{j}$ denotes the matrix obtained by deleting the i-th row and the j-th column from Φ. We say that a sequence of integers $z_1, \cdots, z_n$ is connected if $z_i \leqq z_{i+1} \leqq z_i+1$ for all $1 \leqq i \leqq n-1$ or $n=0$ (i.e. the sequence is empty). The ideal sheaf of a curve X in $\mathbf{P}^3$ is denoted by $\mathcal{J}_X$ and we set $I_{X,n}=H^0(\mathcal{J}_X(n))$, $I_X=\bigoplus_n I_{X,n} \subset R$, where $R=k[x_1, x_2, x_3, x_4]$. For simplicity we abbreviate "arithmetically Buchsbaum" to "a. B.".

§ 1. Preliminaries.

Given a curve X in $\mathbf{P}^3$, we define the basic sequence of X to be the sequence of positive integers $(a; \nu_1, \cdots, \nu_a; \nu_{a+1}, \cdots, \nu_{a+b})$ $(b \geqq 0)$ which satisfies the conditions (1.1), (1.2), (1.3) below and denote it by $B(X)$ (see [**1**; §§ 1, 2]). Let x_1, x_2, x_3, x_4 be generic homogeneous coordinates of $\mathbf{P}^3$ and set $R'=k[x_1, x_2, x_3]$, $R''=k[x_3, x_4]$.