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A generating function of strict Gelfand patterns and some formulas on characters of general linear groups

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Introduction.

Gelfand patterns and strict Gelfand patterns are triangular arrays of non-negative integers satisfying certain conditions. I. M. Gelfand and M. L. Zetlin used Gelfand patterns for the parametrization of the weight vectors of representation spaces of general linear groups [4].

Since then, many mathematicians and physicists have used them in order to study representations of the classical groups and corresponding particles. Still, strict Gelfand patterns have mainly been studied from the combinatorial point of view. However, R. P. Stanley has demonstrated an interesting relation between a generating function of strict Gelfand patterns and the 'most singular' values of the Hall-Littlewood polynomials [14], [10].

Since the Hall-Littlewood polynomial is a fundamental tool for investigating the representations of general linear groups over finite fields and local fields, there must be a strong connection between the Gelfand-Zetlin parametrization and the formula of Stanley.

The initial motivation of this paper was to find a natural deformation of Stanley's formula so that it involves the Gelfand-Zetlin parametrization as a specialization. In the course this searching, we encountered a more important formula, Weyl's character formula, as another specialization of our formula. The following is Weyl's character formula for $GL(n, \boldsymbol{C})$.

$$S_{\lambda}(z_1, z_2, \cdots, z_{n-1}, z_n) = \frac{V_{\lambda+\delta}}{\prod_{j>i}(z_i - z_j)}. \tag{3.2.1*}$$

In the above formula, δ is "the half of sum of positive roots", and V_{β} denotes the "Vandermonde-type determinant" of type β.

The left side of (3.2.1*) is called the Schur function associated with the highest weight λ, which is the character of an irreducible representation of $GL(n, \boldsymbol{C})$. Its actual definition will be shown in Section 2. We often denote