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Maximal toral action on aspherical manifolds $\Gamma \backslash G/K$ and G/H

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Introduction.

In this note, we shall consider only topological actions. For a closed aspherical manifold M, it is well known that if a compact connected Lie group G acts on M effectively, then G is a toral group T^s with $s \leqq$ rank of the center $z(\pi_1(M))$ of the fundamental group $\pi_1(M)$ of M (Theorem 5.6 in [4]). In [5], it was conjectured that if M is a closed aspherical manifold, then

(1) $z(\pi_1(M))$ is finitely generated, say of rank k,

(2) there exists a toral group T^k acting effectively on M.

These have been verified in many cases. For examples, if M is a smooth manifold admitting a Riemannian metric with non-positive sectional curvature or if M is a nilmanifold, then (1) and (2) hold (see [10]).

In this note, we shall prove the following

THEOREM A. *The conjectures* (1) *and* (2) *hold for aspherical manifold of type* $\Gamma \backslash G/K$, *where* G *is a connected non-compact Lie group,* K *a maximal compact subgroup of* G *and* Γ *a torsion free discrete uniform subgroup of* G.

THEOREM B. *The conjectures* (1) *and* (2) *hold for a compact homogeneous aspherical manifold* G/H, *where* G *is a connected non-compact Lie group and* H *a closed subgroup of* G.

In this note, we shall use the following notations;

1. $\boldsymbol{Z}$, $\boldsymbol{R}$ and $\boldsymbol{C}$ denote the ring of integers, the field of real numbers and the field of complex numbers, respectively.
2. $\tilde{G}$ denotes the universal covering of a Lie group G and $\pi: \tilde{G} \to G$ the covering projection.
3. G^0 denotes the identity component of a Lie group G.
4. $z(G)$ denotes the center of a group G.
5. Lie group is assumed to be connected unless the contrary is stated.