J. Math. Soc. Japan
Vol. 40, No. 4, 1988

On some properties on span

Dedicated to Professor Yukihiro Kodama on his 60th birthday

By Kazuhiro KAWAMURA

(Received March 16, 1987)

1. Introduction.

A compact metric space is called a compactum and a continuum means a connected compactum. All maps in this paper are continuous. Lelek [$\mathbf{L}_1$] defined the *span* of compactum X as the following formula.

$$\sigma(X) = \mathrm{Sup}\left\{ \begin{array}{l} c \geqq 0 \mid \text{there exists a continuum } Z \text{ in } X\times X \text{ such that} \\ \qquad p_1(Z)=p_2(Z) \text{ and } d(x, y)\geqq c \text{ for each } (x, y)\in Z \end{array} \right\}$$

$$= \mathrm{Sup}\left\{ \begin{array}{l} c \geqq 0 \mid \text{there exist a continuum } C \text{ and maps } f, g : C\to X \text{ such} \\ \qquad \text{that } f(C)=g(C) \text{ and } d(f(p), g(p))\geqq c \text{ for each } p\in C \end{array} \right\},$$

where p_i denotes the projection $X\times X\to X$ to the i-th factor, $i=1, 2$. A continuum is called *chainable*, if it is represented as an inverse limit of closed intervals. Each chainable continuum has span zero, but it is not known whether the converse implication holds or not. So it is natural to study the following general question due to Duda and Lelek (Continuum theory problems edited by Lewis [**Lw**], Problem 162).

QUESTION. *To what extent does span zero parallel chainability?*

Several results in this direction has been obtained by several authors. In particular, Duda asked, in the above question, whether any open image of a continuum of span zero has span zero. Oversteegen has proved that open maps between hereditarily indecomposable continua preserve span zero [**O**].

The purpose of Section 2 of this paper is to answer the above question in the affirmative. Our main tool is 'Whyburn's section theorem' and (the method of proof of) an extension theorem of open maps due to Mačkowiak and Tymchatyn [**M-T**].

In Section 3, we will define two properties of continua, using span, which are weaker than the property of having span zero. After some simple results, we will study whether these properties are preserved by maps which preserve span zero.