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Flow equivalence of translations on compact metric abelian groups

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1. Introduction.

Let Γ be a countable discrete subgroup of the group $\boldsymbol{T}^1=\{z\in\boldsymbol{C}\,|\,|z|=1\}$. The character group $\Gamma^\wedge$ of Γ is a compact metric abelian group. Let χ_Γ be an element of $\Gamma^\wedge$ determined by $\langle z, \chi_\Gamma\rangle=z$ for $z\in\Gamma$, and $R(\Gamma)$ a homeomorphism of $\Gamma^\wedge$ defined by $R(\Gamma)\chi=\chi\chi_\Gamma$ for $\chi\in\Gamma^\wedge$. $R(\Gamma)$ is called the translation of $\Gamma^\wedge$. The notion of flow equivalence of homeomorphisms was introduced by W. Parry and D. Sullivan [2]. In this article we are concerned with flow equivalence of translations $R(\Gamma)$. This is closely related with stable isomorphism of irrational rotation C^*-algebras (N. Riedel [3], M. Rieffel [4], S. Kawamura and H. Takemoto [1]). We prove the following

THEOREM. *For countable subgroups Γ_1 and Γ_2 of $\boldsymbol{T}^1$, translations $R(\Gamma_1)$ and $R(\Gamma_2)$ are mutually flow equivalent if and only if there exists a positive constant c such that $K_1=cK_2$, where K_j are subgroups of $\boldsymbol{R}$ defined by $K_j=\{x\in\boldsymbol{R}\,|\,\exp(2\pi ix)\in\Gamma_j\}$, $j=1, 2$.*

As an application we shall give necessary and sufficient conditions for flow equivalence of n-dimensional irrational rotations, adding machine transformations and solenoidal transformations respectively in the following examples.

EXAMPLE 1. Let $\lambda(1), \lambda(2), \cdots, \lambda(n)$ be rationally independent irrational numbers and $\Gamma=\{\exp(2\pi i\sum_{j=1}^{n}m(j)\lambda(j))\,|\,m(j)\in\boldsymbol{Z},\ j=1, 2, \cdots, n\}$. The translation $R(\Gamma)$ is topologically conjugate with an n-dimensional irrational rotation $T=T(\lambda(1), \lambda(2), \cdots, \lambda(n))$ defined by $T(x_1, x_2, \cdots, x_n)=(x_1+\lambda(1), x_2+\lambda(2), \cdots, x_n+\lambda(n))$ for $(x_1, x_2, \cdots, x_n)\in\boldsymbol{R}^n/\boldsymbol{Z}^n$. Our theorem implies that irrational rotations $T(\lambda(1), \lambda(2), \cdots, \lambda(n))$ and $T(\mu(1), \mu(2), \cdots, \mu(n))$ are mutually flow equivalent if and only if there exist a positive constant c and a matrix $A\in SL(n+1, \boldsymbol{Z})$ such that

$$(1, \lambda(1), \lambda(2), \cdots, \lambda(n))=c(1, \mu(1), \mu(2), \cdots, \mu(n))A.$$

EXAMPLE 2. Let $r=(r_n)_{n\geq1}$ be a sequence of integers $\geqq 2$, and $\Gamma=\{\exp(2\pi ik/$