J. Math. Soc. Japan
Vol. 40, No. 3, 1988

On the existence of periodic solutions to nonlinear abstract parabolic equations

By Hiroko OKOCHI

(Received Nov. 12, 1985)
(Revised Feb. 18, 1987)

Introduction.

This paper concerns the nonlinear parabolic equation in a real Hilbert space H, which is of the form

$$\frac{d}{dt}u(t)+\partial\varphi(u(t)) \ni f(t), \tag{E}$$

where $f\in L_{\mathrm{loc}}^{2}(\boldsymbol{R}; H)$, φ is a proper l.s.c. (lower semi-continuous) convex functional on H and $\partial\varphi$ is the subdifferential of φ.

The existence of periodic solutions to (E) has been studied by many authors under some assumptions on $\partial\varphi$ and f (see [4], [7], [8], [12]).

The purpose of this paper is to show the existence of anti-periodic solutions to (E) under some condition different from coerciveness. This is motivated by the fact that generally elliptic operators defined on unbounded domains of $\boldsymbol{R}^{n}$ are not coercive. We show the existence of anti-periodic solutions in case $\partial\varphi$ is *odd* (Theorem 1.1). Next we apply this result to a nonlinear heat equation defined on an exterior domain of $\boldsymbol{R}^{n}$ (Section 3). Finally we give examples to see that the conditions assumed in Theorem 1.1 are essential for the existence of a periodic solution to (E) (see Propositions 1.1 and 1.2).

1. Results.

Let H be a real Hilbert space with inner product $(\cdot, \cdot)$ and norm $\|\ \|$. We consider the existence of periodic solutions to the equation;

$$\frac{d}{dt}u(t)+\partial\varphi(u(t)) \ni f(t). \tag{E ; φ, f}$$

Here φ is a proper l.s.c. convex functional on H and $\partial\varphi$ is the subdifferential of φ and $f\in L_{\mathrm{loc}}^{2}(\boldsymbol{R}; H)$.

Let g be a locally square-integrable function on $\boldsymbol{R}$ with values in H. Then