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Diastases and real analytic functions on complex manifolds

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Introduction.

Let M be a complex manifold with the complex structure $\boldsymbol{J}$. A $\boldsymbol{J}$-invariant symmetric tensor g is called a *Kaehler tensor* if the associated 2-form $\omega_g(X, Y) = g(X, \boldsymbol{J}Y)$ $(X, Y \in \mathrm{TM})$ is closed. In addition, if g is non-degenerate it is called an *indefinite Kaehler metric*. A Kaehler tensor is called *analytic* if it is real analytic. Let $\boldsymbol{C}^{r,s}$ be a complex linear space $\boldsymbol{C}^N$ $(N=r+s)$ with the indefinite Kaehler metric:

$$g_{r,s} = 2\left\{\sum_{\sigma=1}^{r} d\xi^{\sigma}\otimes d\bar{\xi}^{\sigma} - \sum_{\sigma=1}^{s} d\xi^{\sigma+r}\otimes d\bar{\xi}^{\sigma+r}\right\},$$

where $(\xi^1, \cdots, \xi^N)$ denotes the canonical complex coordinate system.

E. Calabi [1] gave a necessary and sufficient condition for a Kaehler manifold to be locally immersed into a complex space form as a Kaehler submanifold, and showed the rigidity of such immersions. In this paper we discuss the existence and the local rigidity of a full holomorphic mapping of M into $\boldsymbol{C}^{r,s}$ preserving a Kaehler tensor, and give several applications, where "full" means that the image of the mapping does not lie in any complex hyperplane in $\boldsymbol{C}^{r,s}$.

In §1, we generalize the concept of diastases (introduced by Calabi [1] for analytic Kaehler metrics) for analytic Kaehler tensors and prepare some basic facts. In §2, we define the *rank* of an analytic Kaehler tensor. For a Kaehler tensor of finite rank, a pair of integers, called *"extended signature"*, is introduced. The Calabi condition for a local existence of holomorphic and isometric immersions into $\boldsymbol{C}^{N,0}$ (which is said to be *resolvable of rank N*) coincides with the condition that the extended signature is $(N, 0)$. We prove the following:

THEOREM. *A simply connected complex manifold M with a Kaehler tensor g admits a full holomorphic mapping Φ into $\boldsymbol{C}^{r,s}$ such that $\Phi^* g_{r,s} = g$ if and only if g is analytic and the extended signature is (r, s). Moreover Φ is locally rigid.*

Furthermore we mention some facts about holomorphic mappings into the Hilbert space l^2.