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Rings with only finitely many isomorphism classes of indecomposable maximal Buchsbaum modules

Dedicated to Professor Hiroyuki Tachikawa on his 60th birthday

By Shiro GOTO and Kohji NISHIDA

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1. Introduction.

The purpose of this paper is to prove the following

THEOREM (1.1). *Let* $P=k[\![X_1, X_2, \cdots, X_n]\!]$ *be a formal power series ring over an algebraically closed field* k *of* $\mathrm{ch}\, k\neq 2$. *Let* $R=P/I$, *where* I *is an ideal of* P *and suppose that* $\dim R=d\geqq 2$. *Then the following two conditions are equivalent.*

(1) *R is a regular local ring.*

(2) *R is a Cohen-Macaulay ring that possesses only finitely many isomorphism classes of indecomposable maximal Buchsbaum modules.* (*See Section* 2 *for the notion of maximal Buchsbaum module.*)

When this is the case, the syzygy modules of the residue class field k of R are the representatives of indecomposable maximal Buchsbaum modules and so there are exactly d non-isomorphic indecomposable maximal Buchsbaum modules over R.

Our contribution in the above theorem is the implication (2)⇒(1). The last assertion and the implication (1)⇒(2) are due to [**6**] (see also [**5**, Theorem 3.2]), where some consequences of the result are discussed too.

We would like to note here that the assumption $\dim R\geqq 2$ in Theorem (1.1) is not superfluous. There actually exist non-regular Cohen-Macaulay local rings R of $\dim R=1$ that possess only finitely many isomorphism classes of indecomposable maximal Buchsbaum modules. The typical example is the ring

$$R = k[\![X, Y]\!]/(X^3+Y^2)$$

(k, any field), which has exactly 5 indecomposable maximal Buchsbaum modules (cf. (5.3)). So the result of one-dimensional case seems more complicated.

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