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On local deformations of a Banach space of analytic functions on a Riemann surface

Dedicated to Professor Kôtaro Oikawa on his 60th birthday

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§ 1. Introduction.

Let $\mathcal{S}$ be the set consisting of all compact bordered Riemann surfaces. For $\bar{S}$ in $\mathcal{S}$, we denote its interior and its border by S and ∂S, respectively. Let us denote by p ($\geqq 0$) the genus of $\bar{S}$ and by q ($\geqq 1$) the number of boundary components of $\bar{S}$. We set

$$N=2p+q-1\,.$$

Furthermore we denote by $A(S)$ the set of all functions which are analytic in S and continuous on $\bar{S}$. It forms a Banach space with the supremum norm

$$\|f\|=\sup_{z\in S}|f(z)|\,.$$

For $\bar{S}$ and $\bar{S}'$ in $\mathcal{S}$, let $L(A(S), A(S'))$ denote the set of all continuous invertible linear mappings of $A(S)$ onto $A(S')$. It is shown by Rochberg [6] that $L(A(S), A(S'))$ is nonvoid if S and S' are homeomorphic. We set

$$c(T)=\|T\|\|T^{-1}\|$$

for T in $L(A(S), A(S'))$. We have always

$$c(T)\geqq 1\,,$$

and if $T1=1$, we see that

$$1\leqq\|T\|\leqq c(T)\,,\qquad 1\leqq\|T^{-1}\|\leqq c(T)$$

and that

$$c(T)^{-1}\|f\|\leqq\|Tf\|\leqq c(T)\|f\|$$

for all f in $A(S)$. The above inequality implies that in the case $T1=1$, T is an isometry if and only if the value $c(T)$ attains its minimum 1. Therefore the value $\log c(T)$ is considered to be the quantity representing the deviation of T from isometries. This quantity was first studied by Banach and Mazur for more general cases (cf. [2]). It is well known that if there exists an isometry T in