J. Math. Soc. Japan
Vol. 40, No. 3, 1988

Space curves of genus 7 and degree 8 on a non-singular cubic surface with stable normal bundle

By Tomoaki ONO

(Received Jan. 5, 1987)

Introduction.

D. Perrin showed in [8] that the normal bundles of curves of degree s^2-1 which are linked to a line by two surfaces of degree s in $\boldsymbol{P}^3$ are semi-stable. In the case of $s=3$, the above curves have genus 7 and degree 8. In this paper, we shall show that the normal bundles of general non-singular curves of genus 7 and degree 8 on a non-singular cubic surface in $\boldsymbol{P}^3$ are stable (Theorem (2.3)).

In § 1 we determine divisor classes of non-singular curves of genus 7 and degree 8 on a non-singular cubic surface in $\boldsymbol{P}^3$. In § 2 we evaluate the number of isolated singular points of a cubic surface containing the above curve (Lemma (2.2)). This evaluation plays an important role in the proof of Theorem (2.3). In § 3 we give examples of non-singular curves of genus 7 and degree 8 with non-stable normal bundle. In § 4 we consider a few projectively normal curves on a non-singular cubic surface which are not contained in any quadric surface.

NOTATION. Throughout this paper we shall work over the ground field $\boldsymbol{C}$ and $\boldsymbol{C}^*$ denotes the multiplicative group of $\boldsymbol{C}$. Let X be a non-singular projective variety and let E be a vector bundle on X.

$h^i(X, E) := \dim_{\boldsymbol{C}} H^i(X, E)$; the dimension of $H^i(X, E)$,

$H^i(X, E)^\vee$; the dual vector space of $H^i(X, E)$,

$E^* := \mathrm{Hom}_{\mathcal{O}_X}(E, \mathcal{O}_X)$; the dual vector bundle of E.

Moreover, if C is a curve on a surface S in $\boldsymbol{P}^3$, we use the same symbol C for the corresponding divisor class on S.

I_C; the ideal sheaf of C in $\boldsymbol{P}^3$,

N_C; the normal sheaf of C in $\boldsymbol{P}^3$,

$N_{C/S}$; the normal sheaf of C in S.