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An elementary and unified approach to the Mathieu-Witt systems

Dedicated to Professor Nagayoshi Iwahori on his 60th birthday

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1. Introduction.

Up to now, a great variety of interesting and suggestive studies on the Mathieu-Witt systems $\boldsymbol{W}_{24}$ and $\boldsymbol{W}_{12}$ have been made by many people. In particular, Cameron [**2**, Chapters 2, 3] and Conway [**4**, Section 3] (resp., Beth [**1**]) studied $\boldsymbol{W}_{24}$ (resp., $\boldsymbol{W}_{12}$) using symmetric differences effectively, and Curtis [**5**] (resp., [**6**]) studied them introducing the somewhat magical concepts of the MOG=Miracle Octad Generator (resp., the Kitten). Although these studies have revealed many fascinating facts about the systems, it seems that the essence of them is not yet satisfactorily elucidated — for example, it seems that the treatment of both systems is not sufficiently unified and that the way of describing blocks is not so simple.

The aim of this article is to present a description of both systems from scratch in as orderly, unified and elementary a manner as possible, using mainly symmetric differences and linear fractional groups PSL(2, q). The next section collects some notation (including $\boldsymbol{D}(q, A)$) and facts on symmetric differences, which are used throughout the article. In Sections 3 and 4, in a unified way (via $\boldsymbol{D}(q, A)$) we construct the two systems and an infinite class of 3-$(q+1, (q+1)/2, (q+1)(q-3)/8)$ designs, where q is a prime power with $q\equiv-1 \pmod 4$ and $q>7$. In Section 5, which is the main body of this article, we present a simple and unified way of describing all the blocks of the two systems (and $\boldsymbol{W}_{23}$, $\boldsymbol{W}_{11}$). Namely, (instead of the MOG and Kitten) we introduce a concept of difference patterns or representative blocks, which enables us to enumerate all the blocks uniformly and immediately, and to find quickly the unique block containing five (four) given points.

All the discussions (except the proof of Proposition 3.1) in this article are completely elementary, and a considerable part of them already may be known

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