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Compactness of the moduli space of Yang-Mills connections in higher dimensions

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§1. Introduction and statement of results.

In analytical aspect of the Yang-Mills theory one of the most fundamental results is the K. Uhlenbeck's compactness theorem on the moduli space of Yang-Mills connections.

The purpose of the present paper is to generalize the theorem of Uhlenbeck to higher dimensions. More precisely, let G be a compact Lie group, and $\{D(i)\}$ a sequence of Yang-Mills connections on a G-principal P over an n-dimensional Riemannian manifold M such that for some constant R

$$\int_M |R(i)|^2 dV \leqq R < \infty .$$

Then we can state the theorem of K. Uhlenbeck:

(1.1) FACT ([8], [2]). *Let* $2 \leqq n \leqq 4$. *Then there exist a subsequence* $\{j\} \subset \{i\}$, *a subset* M' $(\subset M)$, *and a Yang-Mills connection* $D(\infty)$ *on* P *over* M' *such that* $M-M'$ *consists of at most finitely many points* $\{p_1, \cdots, p_l\}$, *and that for each compact subset* $K \subset M'$ *there exist gauge transformations* $g_K(j)$ *of* P *over* K *so that*

$$g_K(j)^*(D(j)) \longrightarrow D(\infty) \qquad \text{in } C^\infty\text{-topology on } K.$$

Furthermore,

a) *when* $n=2, 3$, *we have* $M'=M$,

b) *when* $n=4$, *in a neighborhood of each* p_k, *the following happens: If* $x=(x_1, x_2, x_3, x_4)$, $|x|<\delta$, *denote normal coordinates of* M *at* p_k, *then there are rescalings* $\rho(j)(x)=(1/r_j)x$ *of this coordinates with* $r_j \to 0$ *such that for each compact subset* $H \subset \boldsymbol{R}^n$ *there exist gauge transformations* $\gamma_H(j)$ *of* $\rho(j)^*P$ *over* H *so that*

$$\gamma_H(j)^*\rho(j)^*D(j) \longrightarrow D \qquad \text{in } C^\infty\text{-topology on } H$$

where D *is a non-flat Yang-Mills connection on* $\boldsymbol{R}^4$ *with respect to the standard metric of* $\boldsymbol{R}^4$ *with finite action*