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Rings of automorphic forms which are not Cohen-Macaulay, II

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In [2], [3], Eichler posed the question whether or not a ring of automorphic forms (particularly, Hilbert and Siegel modular forms) is Cohen-Macaulay (C.-M. for short). Freitag [4] first gave the negative answer to the question in the case of a ring of Hilbert modular forms of dimension $\geqq 3$. In our previous papers [18], [19] we have surveyed the question, and as for Siegel modular forms we have got the following results. Let $\Gamma_n := Sp_{2n}(\boldsymbol{Z})$, and let $\Gamma_n(l)$ be its congruence subgroup of level l; $\{M \in \Gamma_n \mid M \equiv 1_{2n} \bmod l\}$. For a congruence subgroup Γ of $Sp_{2n}(\boldsymbol{Z})$ let $A(\Gamma)=\bigoplus_{k\geq 0} A(\Gamma)_k$ denote the graded ring of Siegel modular forms for Γ, $A(\Gamma)_k$ being the vector space of modular forms of weight k. Let $A(\Gamma)^{(r)}$ denote the ring $\bigoplus_{k\equiv 0(r)} A(\Gamma)_k$ for an integer r. Then

(i) $A(\Gamma_2(l))^{(r)}$ is not C.-M. for any r if $l \geqq 6$.

(ii) Let Γ be a neat congruence subgroup of $Sp_{2n}(\boldsymbol{R})$ with $n \geqq 3$. Then $A(\Gamma)^{(r)}$ is not C.-M. for any r.

(iii) $A(\Gamma_n)^{(r)}$ is not C.-M. for any r if $n \geqq 4$.

Concerning $A(\Gamma_n)$ $(n \geqq 1)$, it is only a remaining problem if $A(\Gamma_3)^{(r)}$ is C.-M., since $A(\Gamma_1)^{(r)}$, $A(\Gamma_2)^{(r)}$ are known to be C.-M. for any r, or at least it is an easy consequence of the structure theorems of $A(\Gamma_1)$, $A(\Gamma_2)$ (cf. Igusa [12], [13]). In the present paper we show that $A(\Gamma_3)^{(r)}$ *is not* C.-M. *for any* r.

$A(\Gamma_n)$ $(n \geqq 3)$ has been shown to be U.F.D. by Freitag [5], [6] (cf. Tsuyumine [20]), and so they furnish negative examples of the question whether U.F.D. is C.-M. which is posed by Samuel [16]. In the case of characteristic 0, Freitag and Kiehl [7] first gave the negative example to this question (see also S. Mori [15]).

Our method to prove that $A(\Gamma_3)^{(r)}$ is not C.-M. is as follows. If $A(\Gamma_3)^{(r)}$ is C.-M., then the Satake compactification X_3^* of the quotient space H_3/Γ_3 would be a C.-M. variety, and so the Serre duality would hold on it. Then $\dim H^6(X_3^*, \mathcal{O}_{X_3^*})$ must be equal to one since $H^6(X_3^*, \mathcal{O}_{X_3^*})$ is dual to the group of global sections of the coherent sheaf on X_3^* corresponding to modular forms of weight four, and since there is the unique modular form of weight four up to constant multiples. Thus to prove our assertion it is enough to show the