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Construction of the solutions of microhyperbolic pseudodifferential equations

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§ 0. Introduction.

In this paper, we give an explicit representation of the microfunction solutions of microhyperbolic pseudodifferential equations. Microhyperbolicity is an important notion introduced by M. Kashiwara and T. Kawai in [**4**], and such equations were investigated by [**4**] and [**5**]. These authors proved the existence of the solutions in an abstract manner, using a continuation theorem of holomorphic functions in complex domains. Our aim is to construct the solutions explicitly to the contrary. The basic idea of our theory is due to M. D. Bronstein [**3**]. We extend the arguments of [**3**] microlocally in the category of hyperfunctions, and define a very general class of operators, which will be called Bronstein operators (see § 2 for the precise definition). For that purpose, we employ a different formulation from that of [**3**]. We shall show how the arguments of [**3**] can be applied when one considers the defining functions of microfunctions. It will turn out that such an approach is successful, and we can directly construct the solutions of microhyperbolic pseudodifferential equations.

Recently K. Kataoka [**6**] gave an amelioration of our Proposition 4.7 below. He proved a more precise symbol formula for our operator theory extended in § 4. He also suggests that our theory can be understood from a wider point of view, and that it can then be applied to boundary value problems. S. Wakabayashi [**10**] investigated hyperbolic Cauchy problems in detail, also using Bronstein theory. Recently K. Kajitani and S. Wakabayashi [**11**] extended such a theory microlocally. The author thinks that our construction is more direct, although the basic idea is closely connected. They also gave a detailed result in the Gevrey category, and it seems that they also look for applications to boundary value problems.

To state the main theorem, we give some preliminaries. Let $x=(x_1, x')\in \boldsymbol{R}\times\boldsymbol{R}^{n-1}$ or $\boldsymbol{C}\times\boldsymbol{C}^{n-1}$, and let $D=\partial/\partial x$. If $q\in\boldsymbol{Z}_+=\{0, 1, 2, \cdots\}$, $s\in\boldsymbol{R}$ and $s>1$, we denote by $C^q[\boldsymbol{R}; \mathcal{D}^{(s)\prime}(\boldsymbol{R}^{n-1})]$ the space of $\mathcal{D}^{(s)\prime}(\boldsymbol{R}^{n-1})$ valued