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# On the number of exceptional values of the Gauss maps of minimal surfaces

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## §1. Introduction.

In 1961, R. Osserman showed that the Gauss map of a complete non-flat minimal (immersed) surface in $\boldsymbol{R}^3$ cannot omit a set of positive logarithmic capacity ([8]). Moreover, he proved the following:

THEOREM 1.1 ([9]). *Let $M$ be a minimal surface in $\boldsymbol{R}^m$ $(m\geqq 3)$, and $p$ be a point of $M$. If all normals at points of $M$ make angles of at least $\alpha$ with some fixed direction, then*

$$|K(p)|\leqq\frac{1}{d(p)^2}\cdot\frac{16(m-1)}{\sin^4\alpha},$$

*where $K(p)$ and $d(p)$ denote the Gauss curvature of $M$ at $p$ and the distance from $p$ to the boundary of $M$ respectively.*

Afterwards, F. Xavier gave the following improvement of the former result of R. Osserman.

THEOREM 1.2 ([11]). *The Gauss map of a complete non-flat minimal surface in $\boldsymbol{R}^3$ can omit at most six points of the sphere.*

Recently, the author gave a generalization of this to the case of complete minimal surfaces in $\boldsymbol{R}^m$ $(m\geqq 4)$ ([4], [5]). He studied also the value distribution of the Gauss map of a complete submanifold $M$ of $\boldsymbol{C}^m$ in the case where the universal covering of $M$ is biholomorphic to the unit ball in $\boldsymbol{C}^n$ ([6]).

In this paper, relating to these results we shall give the following theorem.

THEOREM I. *Let $M$ be a minimal surface in $\boldsymbol{R}^3$. Suppose that the Gauss map $G: M\rightarrow S^2$ omits at least five points $\alpha_1, \cdots, \alpha_5$. Then, there exists a positive constant $C$ depending only on $\alpha_1, \cdots, \alpha_5$ such that*

$$|K(p)|\leqq\frac{C}{d(p)^2}$$

*for an arbitrary point $p$ of $M$.*