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Singularities of the scattering kernel for two balls

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§ 1. Introduction.

Let $\mathcal{O}$ be a compact obstacle in $\boldsymbol{R}^n$ ($n\geqq 2$) with a C^∞ boundary $\partial\Omega$, and assume that $\Omega=\boldsymbol{R}^n-\mathcal{O}$ is connected. Let us consider the scattering by $\mathcal{O}$ expressed by the equation

$$(1.1)\qquad \begin{cases} \Box u(t, x)=0 & \text{in } \boldsymbol{R}^1\times\Omega \quad (\Box=\partial_t^2-\Delta_x), \\ u(t, x')=0 & \text{in } \boldsymbol{R}^1\times\partial\Omega, \\ u(0, x)=f_1(x) & \text{on } \Omega, \\ \partial_t u(0, x)=f_2(x) & \text{on } \Omega. \end{cases}$$

We denote by $k_-(s, \omega)$ ($k_+(s, \omega)$) $\in L^2(\boldsymbol{R}^1\times S^{n-1})$ the incoming (outgoing) translation representation of the initial data $f=(f_1, f_2)$. The scattering operator $S: k_-\rightarrow k_+$ becomes a unitary operator from $L^2(\boldsymbol{R}^1\times S^{n-1})$ to $L^2(\boldsymbol{R}^1\times S^{n-1})$ (cf. Lax and Phillips [5], [6]), and is represented with a distribution kernel $S(s, \theta, \omega)$:

$$(Sk_-)(s, \omega)=\iint S(s-t, \theta, \omega)k_-(t, \omega)dtd\omega.$$

$S(s, \theta, \omega)$ is called the scattering kernel. Lax and Phillips in [5] showed that the scattering operator S determined the obstacle $\mathcal{O}$ uniquely (cf. Theorem 5.6 of Ch. V in [5]). But, it was not made clear how the analytical properties of S were connected with the geometrical properties of $\mathcal{O}$.

Recently some authors have examined the relation between $\mathcal{O}$ and $S(s, \theta, \omega)$. Majda in [7] has obtained the following results in the case of $n=3$:

$$(1.2)\qquad \operatorname{supp} S(\cdot, -\omega, \omega)\subset(-\infty, -2r(\omega)],$$

$$(1.3)\qquad -2r(\omega)\in \operatorname{sing\,supp} S(\cdot, -\omega, \omega),$$

where $r(\omega)=\min_{x\in\mathcal{O}} x\cdot\omega$. The above results are proved also in the case of $n\geqq 2$ by Soga [12]. Soga [11] and Yamamoto [14] have characterized the convexity of $\mathcal{O}$ with the singularities of $S(s, -\omega, \omega)$: