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The global hypoellipticity of degenerate elliptic-parabolic operators

By Kazuo AMANO

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0. Introduction.

As is well-known, there are vast references on (*local*) hypoellipticity of degenerate elliptic-parabolic operators (cf. [**2**], [**3**], [**8**], [**9**], [**13**], [**14**] and their references). However, one can find only few papers concerned with *global* hypoellipticity. Oleinik and Radkevich [**14**] and Fedii [**3**] proved global hypoellipticity of degenerate elliptic-parabolic operators when $S=\{x : \dim \mathrm{Lie}(x)<d\}$ is either a smooth hypersurface or an isolated point. Fedii [**4**] and Fujiwara and Omori [**6**] found an operator which is globally hypoelliptic but not (locally) hypoelliptic; Amano [**1**] generalized their results. In this paper, we shall show sufficient conditions for global hypoellipticity, which are stated in terms of diffusion and drift vector fields. Our results contain Oleinik and Radkevich's, Fedii's and Fujiwara and Omori's theorems as special cases. We can apply our theorems when S is not a smooth hypersurface, and further, since the proof of theorems essentially depends on a certain type of moderate *a priori* estimates (Proposition 2.1), our results are applicable to a wider class of operators.

Recently, Kusuoka and Stroock [**12**] and Omori [**15**] proved similar theorems; their methods are different from ours. Unfortunately, their results are not applicable to the operators which are not of Hörmander type.

Throughout this paper, Ω is an open set of $\boldsymbol{R}^d$, and

$$P=\sum_{i,j=1}^{d} a^{ij}\partial_{x_i}\partial_{x_j}+\sum_{i=1}^{d} b^i\partial_{x_j}+c(x)$$

is a differential operator with real coefficients satisfying

$$a^{ij}(x)=a^{ji}(x)\in C^\infty(\Omega),\quad b^i(x)\in C^\infty(\Omega),\quad c(x)\in C^\infty(\Omega)$$

and

$$\sum_{i,j=1}^{d} a^{ij}(x)\xi_i\xi_j\geqq 0 \qquad \text{for any} \quad (x, \xi)\in\Omega\times\boldsymbol{R}^d\,,$$

i.e., P is a degenerate elliptic-parabolic operator in Ω. We define vector fields $X_0, X_1, \cdots, X_d$ by