J. Math. Soc. Japan
Vol. 40, No. 1, 1988

A construction of certain 3-manifolds with orientation reversing involution

By Masako KOBAYASHI

(Received Aug. 11, 1986)
(Revised Oct. 1, 1986)

1. Introduction.

In his paper [4], Kawauchi proved that if a closed orientable 3-manifold M admits an orientation reversing involution, then the torsion part of the first integral homology group, $\mathrm{Tor}\, H_1(M; Z)$, is isomorphic to $A \oplus A$ or $Z_2 \oplus A \oplus A$ where A is an abelian group of finite order. Moreover, for any given abelian group G with $\mathrm{Tor}\, G \cong A \oplus A$, there exists a closed orientable irreducible 3-manifold M admitting an orientation reversing involution with $H_1(M; Z) \cong G$. And if M is a closed orientable 3-manifold admitting an orientation reversing involution with $H_1(M; Z) \cong Z_2 \oplus A \oplus A$ where A is an abelian group of odd order, then M must be a connected sum of P^3 and a certain manifold.

In this paper, for the remaining cases, we will prove the following theorems.

THEOREM 1. *For any abelian group* G *with* $\mathrm{Tor}\, G \cong Z_2 \oplus A \oplus A$ *(possibly,* $A=0$*) and* $G/\mathrm{Tor}\, G \neq 0$*, there exists a closed orientable irreducible* 3*-manifold* M *admitting an orientation reversing involution with* $H_1(M; Z) \cong G$.

THEOREM 2. *For any abelian group* $G \cong Z_2 \oplus A \oplus A$ *where* A *is an abelian group of non zero even order, there exists a closed orientable irreducible* 3*-manifold* M *admitting an orientation reversing involution with* $H_1(M; Z) \cong G$.

We refer to [2] and [3] for general definitions and terminology.

2. Proof of Theorem 1.

We identify a 3-sphere S^3 with $R^3 \cup \{\infty\}$, and consider the antipodal map $\tau: S^3 \to S^3$ by $\tau(x, y, z) = (-x, -y, -z)$ $\tau(\infty) = (\infty)$.

LEMMA 3. *There exists a closed orientable irreducible* 3*-manifold* M *admitting an orientation reversing involution with* $H_1(M; Z) \cong Z \oplus Z_2$.

PROOF. Consider a graph T in S^3 as in Figure 1. We choose the graph T so that T contains the origin $0 = (0, 0, 0)$ of S^3 and T is invariant by τ, the