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The linearity question for Abelian groups on translation planes

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1. Introduction.

Let Π denote a translation plane of order q^k with kernel $GF(q)$ and let $\mathcal{G}$ be a collineation group of Π in the translation complement. That is, $\mathcal{G}$ is a subgroup of $\Gamma L(2k, q)$. Normally, $\mathcal{G}$ is *taken* to belong to the linear translation complement while simultaneous disclaimers are made as to the differences between the situations linear and nonlinear.

If $\mathcal{G}$ is nonsolvable then there is a nonsolvable subgroup in the linear translation complement. This usually suffices for the study in question. However, when $\mathcal{G}$ is solvable, the fact that $\mathcal{G}$ may not be linear creates many problems.

In several recent articles, translation planes of order q^2 with kernel $GF(q)$ which admit collineation groups of order q^2 have been studied. In order to apply various analyses of functions on finite fields, the group $\mathcal{G}$ is *required* to be in the linear translation complement.

For a general study, we must therefore consider the following:

LINEARITY QUESTION. *If Π is a translation plane of order $q^s=p^{sr}$ with kernel $GF(q)$ admitting a group $\mathcal{G}$ of order q^s in the translation complement, is $\mathcal{G}$ a subgroup of the linear translation complement?*

If Π is a semifield plane of even order q^2 (for example Desarguesian) which admits a Baer involution then there is a group $\mathcal{G}$ of order q^2 such that $|\mathcal{G}\cap GL(\Pi)|=q^2/2$ or q^2 depending on the kernel.

Hence, in order to study the linearity question in dimension 2, we must make an additional assumption.

In the odd order case, a linear group of order q^2 which acts on translation plane of order q^2 and kernel $GF(q)$ turns out to be Abelian (see e.g. [**3**]). So,

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