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Knotted homology 3-spheres in S^5

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§1. Introduction.

A 3-*knot* will denote the (oriented) isotopy class of a smooth (oriented) submanifold K of the 5-sphere S^5, where K is a homology 3-sphere. When the diffeomorphism type Σ of K is to be emphasized, we call the 3-knot (S^5, K) a Σ-*knot*. A 3-knot (S^5, K) is *simple* if $\pi_1(S^5-K)\cong \boldsymbol{Z}$. Simple Σ-knots are classified by their Seifert matrices (Theorem 2.2), just as simple S^3-knots are ([**12**]). A 3-knot is *decomposable* if it is the connected sum of two 3-knots, both different from the trivial S^3-knot.

In this paper, we consider the following four problems using the classification of simple Σ-knots.

(A) Fixing Σ, can one define a "trivial" knot among Σ-knots?

(B) When is a simple 3-knot decomposable?

(C) Does there exist a fibered 3-knot which is, though decomposable, not the connected sum of two fibered 3-knots, both different from the trivial S^3-knot?

(D) If a simple 3-knot is algebraically fibered, when is it geometrically fibered?

As for Problem (A), we define a trivial Σ-knot to be a simple Σ-knot with *trivial* Seifert matrix, i.e., a Seifert matrix S-equivalent to the zero matrix, for each Σ with zero Rohlin invariant. This trivial Σ-knot is unique (by Theorem 2.2) and characterized by the property that $\pi_i(S^5-K)\cong\pi_i(S^1)$ for all i. Furthermore, if Σ bounds a compact contractible 4-manifold M, then K bounds M embedded in S^5 (§5).

We can answer Problem (B) in terms of Seifert matrices (§3). From this we can derive the following notable fact: If Σ is not diffeomorphic to S^3 and has zero Rohlin invariant, all simple Σ-knots except the trivial Σ-knot are decomposable. As an application, we shall determine when an algebraic 3-knot is decomposable (Theorem 3.4). As a corollary of this, we shall obtain the existence theorem of decomposable algebraic 3-knots (Corollary 3.8) analogous to a result of Michel and Weber [**13**].

We answer Problem (C) affirmatively using a result of Donaldson [**4**] (Example 4.1). Thus the solution of Problem (B) does not apply directly to the