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Characterizations of the $\bar{\partial}$-cohomology groups for a family of weakly pseudoconvex manifolds

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Introduction.

On strongly pseudoconvex manifolds one can know the finiteness or the vanishing of the $\bar{\partial}$-cohomology groups by the method of a priori estimate ([**1**, **5**, **9**]).

In the case of the $\bar{\partial}$-problem on weakly pseudoconvex manifolds, the method of a priori estimate seems to be less powerful (for instance, see [**6**, **7**, **10**, **13**, **14**]).

Let T^n be a complex n-dimensional torus and $\mathrm{Pic}^0(T^n)$ the Picard group, that is, the group of holomorphic line bundles on T^n with Chern class zero. Let $E \in \mathrm{Pic}^0(T^n)$. In [**4**] Grauert showed that there exists a C^∞ weakly plurisubharmonic exhaustion function on E. So we can regard $\mathrm{Pic}^0(T^n)$ as a family of weakly pseudoconvex manifolds.

In this paper we obtain a criterion for the $\bar{\partial}$-cohomology in this family $\mathrm{Pic}^0(T^n)$, using the theory of Diophantine approximation.

It was known that $\mathrm{Pic}^0(T^n)$ is again a complex n-dimensional torus. Concretely we give an isomorphism $i: \boldsymbol{C}^n/\Lambda \cong \mathrm{Pic}^0(T^n)$ in Lemma 1, where Λ is a discrete lattice of rank $2n$ in $\boldsymbol{C}^n$. We define on $\mathrm{Pic}^0(T^n)$ the invariant distance

$$d(E, F) := \min\{\|a-b+c\| \,;\, i(a+\Lambda)=E,\ i(b+\Lambda)=F,\ c \in \Lambda\},$$

where $\|(z_1, \cdots, z_n)\| := \max|z_i|$. The unit element of the group $\mathrm{Pic}^0(T^n)$ is denoted by $\mathbf{1}$. We put

$$Q := \{E \in \mathrm{Pic}^0(T^n) \,;\, E^l = \mathbf{1} \text{ for some } l \geqq 1\}.$$

Using Diophantine approximation on $\{d(\mathbf{1}, E^l) \,;\, l \geqq 1\}$, we define the following subsets of $\mathrm{Pic}^0(T^n)$.

$$\mathcal{P} := \{E \in \mathrm{Pic}^0(T^n) \,;\, \inf_{l>1} \exp(al) d(\mathbf{1}, E^l) > 0 \text{ for any } a>0\}.$$

$$\mathcal{R} := \{E \in \mathrm{Pic}^0(T^n) \setminus Q \,;\, \inf_{l>1} \exp(al) d(\mathbf{1}, E^l) = 0 \text{ for some } a>0\}.$$

$$\mathcal{P}^* := \{E \in \mathrm{Pic}^0(T^n) \,;\, \inf_{l>1} \exp(al) d(\mathbf{1}, E^l) > 0 \text{ for some } a>0\}.$$

$$\mathcal{R}^* := \{E \in \mathrm{Pic}^0(T^n) \setminus Q \,;\, \inf_{l>1} \exp(al) d(\mathbf{1}, E^l) = 0 \text{ for any } a>0\}.$$

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