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A note on l-parts of ray class groups

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1. Notation and the result.

Let l be an odd prime number and let k be an algebraic number field of finite degree. For an integer $i>0$, let ζ_i denote a primitive l^i-th root of unity and put $k_i=k(\zeta_i)$. For an ideal $\mathfrak{a}$ of k, let $k(\mathfrak{a})$ denote the group of elements of k prime to $\mathfrak{a}$ and let $k_{\mathfrak{a}}$ denote the ray number group of k modulo $\mathfrak{a}$, i.e., $k_{\mathfrak{a}}=\{x\in k(\mathfrak{a}) \mid x\equiv 1 \bmod \mathfrak{a}\}$. Further, let $I(\mathfrak{a})$ (resp. $P(\mathfrak{a})$) denote the group of ideals (resp. principal ideals) of k prime to $\mathfrak{a}$, and $P_{\mathfrak{a}}$ the ray ideal group of k modulo $\mathfrak{a}$, i.e., $P_{\mathfrak{a}}=\{(x) \mid x\in k_{\mathfrak{a}}\}$. Moreover let $P'_{\mathfrak{a}}$ (resp. $k'_{\mathfrak{a}}$) denote the group of elements of $P(\mathfrak{a})$ (resp. $k(\mathfrak{a})$) whose order modulo $P_{\mathfrak{a}}$ (resp. $k_{\mathfrak{a}}$) is prime to l. The purpose of this note is to prove the following.

THEOREM. *Assume $\zeta_1\notin k$ and $k_1\neq k_2$. Let*

$$1 \longrightarrow N \longrightarrow M \xrightarrow{g} I/P \longrightarrow 1$$

be an abelian extension of the ideal class group I/P of k by a finite abelian l-group N. Then there exist infinitely many ideals S of k which satisfy the following: there is an isomorphism $\Phi: I(S)/P'_S\to M$ such that Φ induces an isomorphism $\Phi: P(S)/P'_S\to N$ and the diagram

$$\begin{array}{ccccccccc} 1 & \longrightarrow & P(S)/P'_S & \longrightarrow & I(S)/P'_S & \longrightarrow & I/P & \longrightarrow & 1 \\ & & \downarrow \wr\! | & & \downarrow \wr\! | & & \| & & \\ 1 & \longrightarrow & N & \longrightarrow & M & \longrightarrow & I/P & \longrightarrow & 1 \end{array}$$

commutes.

2. Proof of the theorem.

Let $(a_i)_{i=1,\cdots,s}$ and $(b_j)_{j=1,\cdots,r}$ be bases of M and N, respectively. Choose distinct prime ideals $\mathfrak{a}_1, \cdots, \mathfrak{a}_s$ prime to l which represent $g(a_1), \cdots, g(a_s)$, respectively (if $g(a_i)=1$, then choose an arbitrary principal prime ideal $\mathfrak{a}_i$). Put $A=$

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