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Moishezon threefolds homeomorphic to P^3

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Introduction.

A compact complex threefold is called a Moishezon threefold if it has three algebraically independent meromorphic functions on it. The purpose of this article is to prove

THEOREM. *A Moishezon threefold homeomorphic to complex projective space P^3 is isomorphic to P^3 if the Kodaira dimension of it is less than three.*

As a corollary to it, we obtain,

THEOREM. *An arbitrary complex analytic (global) deformation of P^3 is isomorphic to P^3.*

As for the (topological) characterization of P^n, it is known that an arbitrary Kählerian complex manifold homeomorphic to P^n is isomorphic to P^n by Hirzebruch-Kodaira [**9**] and Yau [**24**] (see also [**17**]). However neither of the above theorems are entirely clear from this because both a Moishezon threefold and a complex analytic deformation of a compact Kählerian threefold can be nonKählerian as Hironaka's example shows [**6**]. Recently Tsuji [**23**] claims that he is able to prove the second theorem for P^n, whereas Peternell [**19**] asserts both of the above theorems in a stronger form. However there is a gap in the proof of [**19**], as the author of [**19**] himself admits at the end of the article. After completing this article, I received two preprints of Peternell [**20**], [**21**] via Tsunoda and Nishiguchi, in which Peternell claims that he completes the proof of [**19**]. See (3.3).

In this article, we make an approach different from theirs and give an elementary proof of the above theorems.

Our idea of the proof of the first theorem is as follows. Let X be a Moishezon threefold homeomorphic to P^3 whose Kodaira dimension is less than three. Let L be the generator of $\mathrm{Pic}X\,(\cong \boldsymbol{Z})$ with L^3 equal to one. First we notice that $K_X=-4L$ [8], [**17**] and that $\dim|L|$ is not less than three. For an arbitrary pair D and D' in the complete linear system $|L|$, the scheme-theoretic complete intersection l of D and D' is a pure one dimensional con-