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Anosov maps on closed topological manifolds

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Introduction.

The notion of Anosov maps given in Definition 4 is a strict generalization of expanding maps and expansive homeomorphisms with pseudo-orbit tracing property (abbrev. POTP). If in particular an Anosov map is bijective, then it has expansivity and POTP (see Remark 2 (i)). But we remark that the notion of expanding maps is not defined for homeomorphisms of compact connected metric spaces which are not one point (see Remark 2 (ii) (b)). It is known (cf. A. Morimoto [**10**]) that every homeomorphism with expansivity and POTP is topologically stable in the class of homeomorphisms. However it is impossible that a homeomorphism is topologically stable in the class of continuous surjective maps (see Remark 1). By using the same technique in [**10**] we can see that every expanding map satisfies topological stability in the class of continuous surjective maps. Thus it is natural to ask whether every Anosov map which is not bijective satisfies it in the class of continuous surjective maps.

We prove the following

THEOREM 1. *Let M be a closed topological manifold and $f: M \to M$ be a local homeomorphism but not bijective. If f is an Anosov map which satisfies topological stability in the class of continuous surjective maps, then f must be expanding.*

Let (X, d) be a compact metric space and $f: X \to X$ be a continuous surjective map. Denote by Ω the non-wandering set of f ($\Omega \neq \emptyset$ since X is compact). We obtain Smale-Bowen's decomposition theorem for an Anosov map of X as follows.

THEOREM 2. *Every Anosov map f of X has the following properties.*

(i) *$f(\Omega)=\Omega$ and $f: \Omega \to \Omega$ is an Anosov map,*

(ii) *Ω contains a finite sequence B_i ($1 \leqq i \leqq l$) of f-invariant (i.e. $f(B_i)=B_i$) closed subsets such that $\Omega=\bigcup_{i=1}^{l} B_i$ and $f: B_i \to B_i$ is topologically transitive,*

(iii) *for $1 \leqq i \leqq l$, there exists $a>0$, and B_i contains a finite sequence C_{i_j} ($0 \leqq j \leqq a-1$) such that $f^a(C_{i_j})=C_{i_j}$, $C_{i_j} \cap C_{i_{j'}}=\emptyset$, $f(C_{i_j})=C_{i_{j+1}}$ for $0 \leqq j \neq j' \leqq a-1$ ($C_{i_a}=C_{i_0}$), $f^a: C_{i_j} \to C_{i_j}$ is topologically mixing and $B_i=\bigcup_{j=0}^{a-1} C_{i_j}$.*