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Finite subgroups of mapping class groups of geometric 3-manifolds

Dedicated to Professor Itiro Tamura on his 60th birthday

By Ken'ichi OHSHIKA

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Introduction.

In Ohshika [4], we introduced the concept of Teichmüller spaces of Seifert fibered manifolds and studied its properties. In some aspects, they are analogous to Teichmüller spaces of surfaces. Let G be a finite subgroup of the mapping class group of a surface. G acts on the Teichmüller space by pulling back of metrics. In Kerckhoff [2] it is proved that G has a fixed point in the Teichmüller space. We prove that the same theorem holds for Seifert fibered manifolds under some assumptions (Theorem 2.3).

This theorem has an application similar to that of Kerckhoff [2]. It is so-called Nielsen realization problem. For a manifold M, the Nielsen realization problem asks when a finite subgroup G of $\pi_0\mathrm{Diff}^+(M)$ is realized by a group of diffeomorphisms. In dimension 3 there are results of Zimmermann and Zieschang [10], [11], [12] which reduces the problem to algebra on G. On the other hand, for a hyperbolic surface it is proved by Kerckhoff [2] that G can be realized by a group of isometries with respect to some hyperbolic structure. Also for a Haken hyperbolic 3-manifold, G can be realized by a group of isometries, which is proved easily using Mostow's rigidity theorem.

In this paper we deal with Seifert fibered manifolds whose base orbifolds are either hyperbolic or Euclidean. They have geometric structures modelled on one of $H^2\times\boldsymbol{R}$, $\widetilde{SL_2}$, E^3, Nil. We ask if G can be realized by a group of isometries with respect to some geometric structure. We do not use the results of [10], [11], [12]. In §3 we characterize an isometry with respect to a geometric structure which is isotopic to the identity. Using Theorem 2.3 and a proposition in §3, we can solve the realization problem under some assumptions on M and G.

Throughout this paper we work in C^∞ category. All 3-manifolds are assumed to be compact orientable. $\mathrm{Diff}^+(M)$ denotes the group of all orientation preserving diffeomorphisms of M. $\pi_1^{\mathrm{orb}}(O)$ denotes the fundamental group of O as an