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Construction of Hopf G-spaces

Dedicated to the memory of late Professor Shichirô Oka

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§1. Introduction.

Let G be a topological group. The notion of a Hopf G-space is first noted by G. E. Bredon [**3**]. He defined a Hopf G-space to be a space which has a G-equivariant multiplication. Some people did their works in this area, K. Iriye [**6**] on Hopf Z_2-spheres, G. Triantafillou [**11**] on rational cases, etc.

In this paper we shall construct some examples of equivariant Hopf spaces by a method analogous to Zabrodsky's. Actually, A. Zabrodsky exploited his mixing homotopy method ([**13**], [**14**] and [**15**]) to obtain many non-classical Hopf spaces (including the Hilton-Roitberg's example, etc.). We shall discuss an equivariant version of his method under some conditions. For this, we shall use the equivariant localization of J. P. May, et al. [**9**].

Our main results are the following two theorems. Throughout the paper, we assume that G is a compact Lie group.

THEOREM 1.1. *Let S^n be the n-sphere with n odd >1, on which G acts desuspendably, i.e., the action is the suspension of a G-action on S^{n-1}. Let E be a compact Lie group on which G acts by automorphisms. Moreover assume that E acts on S^n transitively and the induced fibration $\pi: E\to S^n$ is a G-fibration, i.e. π is a G-map and has a G-homotopy covering property. Let $h_\lambda: S^n\to S^n$ be the G-map which is of degree $\lambda\in Z$, that is, h_λ is λ times the identity map of S^n in $[S^n, S^n]_G$, and T be a collection of prime numbers. Then the pull back W_{h_λ} in the following diagram*

$$\begin{array}{ccc} W_{h_\lambda} & \longrightarrow & E \\ \big\downarrow & & \big\downarrow \pi \\ S^n & \xrightarrow{\ h_\lambda\ } & S^n \end{array}$$

has a Hopf G-structure, if the following three conditions are satisfied.

a) *E^H and $(S^n)^H$ are connected, and $(S^n)^H$ is also a sphere, for each closed subgroup H of G.*