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Stability of foliations of 3-manifolds by circles

Dedicated to Professor Itiro Tamura on his 60th birthday

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Introduction.

Let $\mathrm{Fol}_q(M)$ denote the set of codimension q C^∞-foliations of a closed m-manifold M. $\mathrm{Fol}_q(M)$ carries a natural weak C^r-topology ($0 \leqq r \leqq \infty$), which is described in [5]. We denote this space by $\mathrm{Fol}_q^r(M)$. We say a foliation F is C^r-stable if there exists a neighborhood V of F in $\mathrm{Fol}_q^r(M)$ such that every foliation in V has a compact leaf. We say F is C^r-unstable if not. We simply say F is (un-)stable if F is C^1-(un-)stable. It seems to be of interest to determine if F is C^r-stable or not.

Let L be a compact leaf of F. Thurston [13] and Langevin-Rosenberg [6] showed, generalizing the Reeb stability theorem [9] that if $H^1(L; \boldsymbol{R})=0$, then F is stable. Let $\pi_1(L) \to GL(q, \boldsymbol{R})$ be the action determined by the linear holonomy of L, where q is the codimension of F. Then generalizing the results of Hirsch [5] and Thurston [13], Stowe [12] showed that if the cohomology group $H^1(\pi_1(L); \boldsymbol{R}^q)$ is trivial, then F is stable.

Let F be a foliation of an orientable S^1-bundle over a closed surface B by fibres. Seifert [11] showed that F is C^0-stable if $\chi(B) \neq 0$, where $\chi(B)$ is the euler characteristic of B. The result was generalized by Fuller [4] to orientable circle bundles over arbitrary closed manifolds B with $\chi(B) \neq 0$. Let $\pi: M \to B$ be a fibration with fibre L. Langevin-Rosenberg [7] showed that the foliation of M by fibres is C^0-stable provided that 1) $\pi_1(L) \cong \boldsymbol{Z}$, 2) B is a closed surface with $\chi(B) \neq 0$ and 3) $\pi_1(B)$ acts trivially on $\pi_1(L)$. The author [3] generalized the above result to compact codimension two foliations. Furthermore Plante [8] gave a necessary and sufficient condition for a transversely orientable foliation of a closed 3-manifold by closed orientable surfaces to be C^0-stable.

We study here the stability of all foliations of closed 3-manifolds by circles and give a necessary and sufficient condition for such a foliation to be stable. Indeed, we have the following theorem.

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