J. Math. Soc. Japan
Vol. 39, No. 1, 1987

On Itô's formula for certain fields of geometric objects

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(Received Oct. 20, 1984)
(Revised July 1, 1985)

1. Introduction.

In stochastic differential geometry, Itô's formula has been extended for tensor fields on a C^∞ manifold in connection with stochastic Lie transport ([**6**], [**11**], cf. [**2**]), stochastic parallel displacement ([**8**], [**11**]), and stochastic flow of tensors induced by tensor derivations ([**1**]).

The purpose of the present paper is to establish a general stochastic formula for C^∞ (local) cross sections of fiber bundles which implies a further extension of Itô's formula to the case of "geometric object fields". As to bundles of geometric objects, we adopt a constructive approach due to Ferraris, Francaviglia and Reina [**3**], [**4**]; thus, by a geometric object field, we mean a C^∞ (local) cross section of a C^∞ fiber bundle associated with the bundle of r-th order frames (over a C^∞ manifold) for some positive integer r. (For a general theory of geometric objects, we refer to Salvioli [**14**]. See also Nijenhuis [**13**], Yano [**15**].) Then, for example, C^∞ tensor fields, C^∞ pseudo-tensor fields, their jet extensions, and linear connections of a C^∞ manifold are geometric object fields. It should be pointed out that the use of frames of higher order contact enables us to treat some geometric structures (for instance, projective structures [**9**]) as geometric object fields.

Now let $\pi_E: E \to M$ be a C^∞ fiber bundle associated with a principal fiber bundle $P(M, G, \pi_P)$ (cf. [**10**]). Let $\varphi_t(p)$ be the solution of the following stochastic differential equation (in the Stratonovich form) on P;

$$d\varphi_t = \sum_{\alpha=1}^{k} A_\alpha(\varphi_t)\circ dN^\alpha(t), \qquad \varphi_0 = p \in P, \tag{3.1}$$

where A_α, $(\alpha=1, \cdots, k)$, are right G-invariant C^∞ vector fields on P and $N^\alpha(t)$'s are real valued continuous semi-martingales. Suppose we are given a C^∞ (local) cross section σ of E. In this paper, we establish a formula for the $\pi_E^{-1}(x)$-

This research was partially supported by Grant-in-Aid for Scientific Research (No. 59740103), Ministry of Education, Science and Culture.