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On automorphism groups of a curve as linear groups

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Introduction.

Let X be a complete non-singular curve over an algebraically closed field k. Assume that G is a finite group of automorphisms of X. Let χ_q $(q=1, 2, \cdots)$ denote the character $\mathrm{Tr}(G|H^0(X, \Omega_X^{\otimes q}))$ of the natural representation of G on the space of q-differentials on X. G or χ_q's have recently been studied with a new importance from their relation with the problems of moduli or Teichmüller space (cf. e.g., [**5**], [**6**], [**11**]).

In the present paper, we confine ourselves to the study of the characters χ_q in the case where G is cyclic and k is of characteristic zero or $k=\boldsymbol{C}$. We attempt to follow up some part of [**5**] and [**8**]. In fact, our aims are (i) to correct a "theorem" in [**5**] concerning the interrelation between the characters, (ii) to reveal a nature of the sequence $(\chi_q)_{q\geq 1}$, and (iii) to characterize $(\chi_q)_{q\geq 1}$ as a sequence of class functions of G by a special type of mapping $\lambda: G\to \mathrm{Map}(\boldsymbol{Z}, \boldsymbol{Q})$.

We shall give a brief survey of this paper. In §1, we shall introduce for G a surjective group homomorphism $\phi_G: \Gamma\to G$, where Γ is a group characterized by the Riemann-Hurwitz relation for the covering: $X\to X/G$. Then we shall give an existence theorem of a cyclic automorphism group in a formulation including ϕ_G (Theorem 1.6). Our basic tool to investigate the characters is the trace formula which says that each χ_q (considered as an unknown) is determined by the information of the homomorphism ϕ_G (cf. (2.1)). In §2, as for (i) we shall show that ϕ_G and hence all of χ_q are determined by the first finite number of the χ_q's (Theorem 2.2). In spite of the importance of χ_1 or χ_2 (for example, χ_2 determines the moduli space near the corresponding point of X), it will be shown (cf. (2.5)) that χ_1 and χ_2 do not necessarily determine other χ_q's (cf. [**5**, p. 219 Corollary]). In §3, as for (ii) we shall prove:

THEOREM. *Let G (resp. G') be a cyclic group of automorphisms of a compact Riemann surface X (resp. X') of genus $\tilde{g}\geqq 2$. Assume that $': G\to G'$ $(\sigma\to\sigma')$ is an isomorphism. Then the following conditions are equivalent.*

(a) *There exists an orientation-preserving homeomorphism $h: X\to X'$ such that*