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A linear prediction problem for symmetric α-stable processes with $1/2<\alpha<1$

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§1. Introduction.

Linear prediction problems of weakly stationary processes are well studied when the processes have second moments [2], [4]. For strictly stationary processes with first moments, Urbanik introduces a class which "admits a prediction" and proves parallel results [9]. In this paper we consider a class of processes which have infinite first moments. Process which admits a prediction is the stationary case of the linear processes that P. Lévy introduced as an extension of the class of Gaussian processes by imposing the linear regression property only on regression on the past [5]. Here we study the case where process X_t has a symmetric α-stable, $0<\alpha<1$, distribution. This is also an example of the class of infinitely divisible processes of Maruyama [6]. We prove that when it is completely non-deterministic it has a canonical representation

$$X_t=\int_{-\infty}^{t}f(t-u)M(du)$$

where $M(du)$ is a stochastic measure such that $\{M(du), u\leqq t\}$ has the same information as $\{X_u ; u\leqq t\}$. We call $M(du)$ innovations of the process X_t. Precise meanings are explained in the following sections.

One of difficulties in our case lies on how to take innovations out of the process. Urbanik uses theories of Banach spaces such as theories of Bochner integral and linear functionals. The linear space spanned by an $S\alpha S$ process, $0<\alpha<1$, is only a Fréchet space. We overcome this difficulty under some additional conditions. In §3, we define a Riemann type integral of functions with values in this Fréchet space and give a sufficient condition for integrability. In §4 we give several lemmas. Although they are similar to Urbanik's lemmas, technique to prove them is quite different from the Banach space case and more complicated. We use the integral of §3 to take innovations out of the processes. Theorems are stated in §5. Depending on our sufficient condition for integrability, we get results only in the case $1/2<\alpha<1$. If we can improve our condition of integrability, we may extend the results to the case $\alpha\leqq 1/2$. This is left for