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A certain class of infinite dimensional diffusion processes arising in population genetics

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1. Introduction.

Let S be a countable set, and let

$$X=\{x=(x_i)_{i\in S} : x_i\geqq 0\ (i\in S),\ \sum_{i\in S} x_i=1\}$$

be the totality of probability vectors on S, which is equipped with the weak topology. Suppose that we are given a second order differential operator L of the following type:

$$L=\frac{1}{2}\sum_{i\in S}\sum_{j\in S}(x_i\beta_i\delta_{ij}+x_ix_j(\sum_{k\in S}x_k\beta_k-\beta_i-\beta_j))\frac{\partial^2}{\partial x_i\partial x_j}+\sum_{i\in S}b_i(x)\frac{\partial}{\partial x_i}, \tag{1.1}$$

where $(\beta_i)_{i\in S}$ are non-negative constants satisfying that $\sup_{i\in S}\beta_i<+\infty$, δ_{ij} stands for the Kronecker symbol, and the domain $\mathcal{D}(L)$ of L is the set of all C^2-functions defined on X depending on only finitely many coordinates.

Let $W=C([0, \infty)\to X)$ be the space of all continuous functions $w: [0, \infty)\ni t\to w(t)\in X$ with the topology of uniform convergence on bounded intervals, and let $\mathcal{F}$ ($\mathcal{F}_t$) be the σ-field on W generated by cylinder sets (up to time t).

By an (X, L)-diffusion we mean a system $\{P_x, x\in X\}$ of probability distributions on $(W, \mathcal{F})$ that is strongly Markovian and satisfies the following two conditions:

$$P_x\{w: w(0)=x\}=1 \quad \text{for every } x\in X, \tag{1.2}$$

$$f(w(t))-f(w(0))-\int_0^t Lf(w(s))ds \text{ is a } (P_x, \mathcal{F}_t)\text{-martingale for every } f\in\mathcal{D}(L). \tag{1.3}$$

In order to construct an (X, L)-diffusion we need boundary conditions and a regularity condition on the drift coefficients $(b_i(x))_{i\in S}$.

ASSUMPTION [B]. $(b_i(x))_{i\in S}$ are real functions defined on X which satisfy the following three conditions:

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