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Traces on group extensions and C^*-crossed products

Dedicated to Professor O. Takenouchi on his 60th birthday

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Introduction.

This paper is devoted to the study of representations, ideals, weights, and especially traces on a C^*-crossed product induced from those on a smaller C^*-crossed product. The main result is the characterization of a trace on a C^*-crossed product $C_r^*(A, G, \alpha)$ to be induced from a trace of a sub-crossed product $C_r^*(A, N, \alpha)$ with N a closed normal subgroup of G.

Let (A, G, α) be a C^*-dynamical system. When N is a closed normal subgroup, starting from certain relatively invariant traces on $C_r^*(A, N, \alpha)$, we have induced traces on $C_r^*(A, G, \alpha)$ [**16**], [**4**]. They have some special properties. When G/N is abelian, they are invariant under the dual action [**16**]. When G/N is not abelian, we have no more a convenient dual action, and we replace the role of the dual action by coaction. As our main result it will be shown that induced traces are characterized by the invariance under the coaction of G/N dual to the original action of G. The statement established may be regarded as an analogue of Takesaki's imprimitivity theorem with respect to traces.

In the theory of ideals of C^*-crossed products, Effros and Hahn have made a famous conjecture. A complete answer to this conjecture has been established in [**3**]. A further conjecture was made by P. Green that Effros-Hahn conjecture would hold for traces. Precisely stating, this conjecture is as follows. "Let (A, G, α) be a C^*-dynamical system, and the action of G on Prim(A) be free. Then all the traces on $C_r^*(A, G, \alpha)$ are induced from A". When G is a discrete amenable group, this was proved in [**4**]. But When G is not discrete, by the absence of conditional expectations, the situation becomes difficult. We believe that the result we obtained here makes a step toward the solution of the problem.

We are given much freedom by the C^*-induction in the studies of induced representations, and the imprimitivity theorem becomes a stronger weapon when it is formulated in the C^*-theoretic framework [**18**]. We use it to obtain a duality for induced representation in non-abelian cases.