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On the explicit models of Shimura's elliptic curves

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Introduction.

In Shimura [11], an abelian variety A over $\boldsymbol{Q}$ is constructed from a "Neben"-type eigen cusp form in $S_2\left(\Gamma_0(N), \left(\frac{}{N}\right)\right)$ for a prime number N such that $N\equiv 1$ mod 4. There is an abelian subvariety B of A rational over $k_N=\boldsymbol{Q}(\sqrt{N})$; they are closely related with the construction of class fields over k_N (Shimura [10]). Moreover, it is known that they have everywhere good reduction over k_N as one of their interesting properties (Deligne-Rapoport [1]). When $N=29$, 37 or 41, they are uniquely determined (so we denote them by A_N and B_N), and B_N is an elliptic curve. On the other hand, some explicit models of elliptic curves with everywhere good reduction over k_N are known (see 1.2). Recently, T. Nakamura has shown that B_{29} is actually isogenous to one of such models ([5], Corollary).

The purpose of this paper is to determine the isomorphism class over k_N of B_N for $N=29$, 37 and 41 (see Theorem 1.3). This can be achieved by calculating the period lattice and the j-invariant of B_N. As a Corollary, we can show the existence of a $\boldsymbol{Q}$-rational point of certain order on A_N (see Corollary 1.4). In Appendix, we shall give a characterization of B_{37}.

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§ 1. Main theorem.

1.1. NOTATION. Let N be a prime number 29, 37 or 41, $\chi(\ )=\left(\frac{}{N}\right)$ the Legendre symbol, and

$$\Gamma=\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix}\in\Gamma_0(N)\ \middle|\ \chi(a)=1\right\}.$$

Denote by X_0 (resp. X) the modular curve which corresponds to $\Gamma_0(N)$ (resp. Γ), and by J_0 (resp. J) its Jacobian variety; X_0, X, J_0, J and the natural homomorphism $J_0\to J$ are all defined over $\boldsymbol{Q}$. Put $A_N=\mathrm{Coker}(J_0\to J)$. Then A_N is a 2-dimensional abelian variety defined over $\boldsymbol{Q}$ which is attached to the Neben-type