

Two intrinsic pseudo-metrics with pseudoconvex indicatrices and starlike circular domains

Dedicated to Professor Tadashi Kuroda on his 60th birthday

By Kazuo AZUKAWA

(Received March 29, 1985)

Introduction.

It is well-known ([7]) that at each point p of a complex manifold M the indicatrix of the *Carathéodory pseudo-metric* C^M of M is always a convex circular domain in the holomorphic tangent space $T_p(M)$ at p . By using a family of bounded plurisubharmonic functions on a complex manifold M , Sibony [19] also defined a biholomorphically invariant pseudo-metric on M whose indicatrices are always convex. On the other hand, Suzuki [21] and Barth [4] showed, independently, that if M is a pseudoconvex starlike circular domain with center at the origin 0 in \mathbf{C}^m , an m -dimensional complex Euclidian space, then the indicatrix of the *Kobayashi pseudo-metric* K^M of M at 0 coincides with M , the tangent space $T_0(M)$ being identified with \mathbf{C}^m in the natural manner. In particular, the indicatrix of K^M at 0 is pseudoconvex. It seems that indicatrices of the Kobayashi pseudo-metric K^M for a complex manifold M are, in general, not necessarily pseudoconvex, for the proofs of the result of Suzuki and Barth mentioned above essentially depend on the pseudoconvexity of the original domain. The main purpose of this paper is to construct, intrinsically, two biholomorphically invariant pseudo-metrics, denoted by B^M and P^M , on a complex manifold M , for each of which the indicatrix at each point is always pseudoconvex in the tangent space.

The paper is organized as follows. In §1, we recall some results concerning starlike circular domains and prove that if M is a pseudoconvex bounded starlike circular domain in \mathbf{C}^m with a continuous boundary, then the Bergman metric of M is complete (Theorem 1.11). Since the boundary of a pseudoconvex Reinhardt domain is continuous, Theorem 1.11 is a generalization of the following theorem of Skwarczyński [20; Theorem 3.16]: The Bergman metric of a pseudoconvex bounded Reinhardt domain in \mathbf{C}^m is complete. Recently, the theorem of Skwarczyński was also improved by Pflug [16]. He proved that the Carathéodory metric of a pseudoconvex bounded Reinhardt domain in \mathbf{C}^m is