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Smooth perturbations in ordered Banach spaces and similarity for the linear transport operators

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1. Introduction.

The aim of this paper is to develop a new method for establishing the similarity for a pair of linear operators in an ordered Banach space X over $\boldsymbol{C}$. Consider a pair of (in general unbounded) linear operators B_1 and $B_2=B_1+A$ with A a bounded operator, and assume that both $-B_1$ and $-B_2$ generate the bounded C_0-groups on X. Assume, in addition, that e^{-tB_1} and $-A$ are positivity preserving, and that A is $(-iB_1, 1)$-smooth. Then we show that B_2 is similar to B_1 (see Theorem 1 in section 2). The similarity of B_2 to B_1 is established by constructing both the wave operator $W_+(B_2, B_1)=\text{s-lim}_{t\to+\infty}e^{tB_2}e^{-tB_1}$ and the inverse wave operator $W_+(B_1, B_2)=\text{s-lim}_{t\to+\infty}e^{tB_1}e^{-tB_2}$. To do this, we simply use Cook's method. Our technique depends heavily upon both e^{-tB_1} and $-A$ being positivity preserving.

There is some literature on the theory of smooth perturbations. Kato [**7**] dealt with a perturbed operator of the form $T(\kappa)=T+\kappa V$ (κ being a small complex-parameter) in a Hilbert space and established the similarity of $T(\kappa)$ to T. Kato's result has been extended to a reflexive Banach space setting by Lin [**9, 10**], and to a not necessarily reflexive Banach space setting by Evans [**3**]. These authors need to factorize the perturbation V into the form D^*C, where C is T-smooth and D is T^*-smooth. We do not use such factorization, however.

Our theory of smooth perturbations is applicable to the linear transport operator (Boltzmann operator) in multiple scattering problem. We consider the linear transport operator

$$(-Bu)(x, \xi) = -\xi\cdot\nabla_x u(x, \xi)-\sigma(x, \xi)u(x, \xi)$$
$$+\int_{\boldsymbol{R}^d}k(x, \xi', \xi)u(x, \xi')d\xi' \qquad (x\in\boldsymbol{R}^d,\ \xi\in\boldsymbol{R}^d)$$

as a perturbation of the collisionless transport operator $-B_0=-\xi\cdot\nabla_x$. Here σ

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