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Limit theorems for point processes and their functionals

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1. Introduction.

The classical limit theorems for sums of independent random variables ([6]) have been extended in several directions. For instance, Skorohold ([19]) discussed functional limit theorems in which sums of independent random variables in a suitable time scale converge to a Lévy process, i. e., a process with independent increments which is continuous in probability. Further, these theorems have been extended to the case of sums of dependent random variables (see e. g. [5]). A unified approach to these problems has been given recently by several authors in the framework of semimartingales. Semimartingales extend the notion of Lévy processes and such basic processes as Wiener processes (i. e. Gaussian martingales) and Poisson point processes are simply characterized and naturally extended in the context of semimartingales.

The purpose of this paper is to discuss limit theorems in the framework of semimartingales represented by stochastic integrals of point processes: We discuss on the convergence of point processes and their functionals defined by stochastic integrals. Similar problem was discussed by several authors (e. g. [5], [8], [10], [16] and [17]), but a main difference is that, in our approach, we do not necessarily assume that the point processes are defined by jumps of semimartingales: Rather, we start with a sequence of point processes and their functionals represented by stochastic integrals and discuss the convergence of them. Our results, of course, overlap those of the above authors but we believe that our proofs are simpler in several points, and it should be remarked that not only Gaussian martingales and Poisson processes but also the general Lévy processes appear in our limit theorems. Also a merit of our approach seems to be in the point that it is useful to clarify the joint convergence of several processes related to a given sequence of point processes. For example, in the case of weighted sums of triangular arrays of random variables, it seems more natural to start with the point processes defined by the original arrays rather than those defined by the weighted sums; we can then consider different weighted sums

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