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Countable models and unions of theories

By Akito TSUBOI

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§ 0. Introduction.

This paper is a natural continuation of [7], in which we gave a partial positive solution to the following conjecture:

CONJECTURE. There is no (1-st order) stable theory which has only n (>1) countable models.

More precisely, in [7], we showed that we cannot construct a theory with n (>1) countable models by imitating the construction of Ehrenfeucht's example. In the present paper, we shall prove the above conjecture for those theories which can be conceived as limits of certain theories.

In § 1, we shall explain some necessary definitions and conventions. Some basic facts are given without proofs.

In § 2, we shall deal with those theories which are unions of ω-categorical theories. The main theorem of § 2 is the following:

THEOREM A. *Let T be the union of ω-categorical theories T_n $(n<\omega)$ such that $T_n \subset T_{n+1}$ $(n<\omega)$. If T has only n (>1) countable models, then T has a (definable) dense order.*

Theorem A is proven by combining the methods used in [4] and [7]: The most important fact used in the proof is that T has a definable dense order if and only if there is a subtheory T_0 of T formulated in some finite language which has a definable dense order. From Theorem A, we can deduce that no ω-categorical theories without dense orders can be extended to theories which have finitely many (>1) countable models by adding axioms for constant symbols.

In § 3, we shall prove the following result for those theories which are unions of pseudo-superstable theories (see Definition 2.1).

THEOREM B. *Let T be the union of pseudo-superstable theories T_n $(n<\omega)$ such that $T_n \subset T_{n+1}$ $(n<\omega)$. Then $I(\omega, T)=1$ or $I(\omega, T)\geqq\omega$.*

This result can be proven by a close examination of Pillay's proof (see [5]) of Lachlan's theorem.