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Spectral geometry of Kaehler submanifolds of a complex projective space

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§ 0. Introduction.

Let $X: M \to E^N$ be an isometric immersion of a compact Riemannian manifold into an N-dimensional Euclidean space. Then X can be decomposed as $X = \sum_{k \in N} X_k$, where X_k is the k-th eigenfunction of the Laplacian of M (for details, see § 2). We say that the immersion is of order $\{k_1, k_2, k_3\}$ (resp. $\{k_1, k_2\}$ and k_1) if $X = X_0 + X_{k_1} + X_{k_2} + X_{k_3}$ (resp. $X = X_0 + X_{k_1} + X_{k_2}$ and $X = X_0 + X_{k_1}$), where X_0 is a constant mapping and $X_{k_1}, X_{k_2}, X_{k_3} \neq 0$ and $0 < k_1 < k_2 < k_3$.

Let $F: CP^m \to E^N$ be the standard isometric imbedding of a complex projective space into an N-dimensional Euclidean space (for details, see § 1), and let $A: M \to CP^m$ be an isometric immersion of a compact Kaehler manifold into an m-dimensional complex projective space. Then A is said to be of order $\{k_1, k_2, k_3\}$ (resp. $\{k_1, k_2\}$ and k_1) if the immersion $F \circ A$ is of order $\{k_1, k_2, k_3\}$ (resp. $\{k_1, k_2\}$ and k_1). A totally geodesic Kaehler submanifold of CP^m is of order 1. Moreover there does not exist any compact Kaehler submanifold of order k_1 ($k_1 \geqq 2$) (see, [8], [9]), and a compact Kaehler submanifold is of order 1 if and only if it is totally geodesic. A. Ros ([9]) proved that Einstein Kaehler submanifolds with parallel second fundamental form except $E_6/Spin(10) \times T$ in a complex projective space are of order $\{1, 2\}$, and he characterized them by their spectra in the class of compact Kaehler submanifolds in a complex projective space. In § 4, we calculate the eigenvalues of the Laplacians of $E_6/Spin(10) \times T$ and $E_7/E_6 \times T$. Consequently, we see that $E_6/Spin(10) \times T$ is of order $\{1, 2\}$, and we can say that a compact Kaehler submanifold different from a totally geodesic Kaehler submanifold in a complex projective space is of order $\{1, 2\}$ if it is Einstein and has parallel second fundamental form (Proposition 3). Moreover we can characterize $E_6/Spin(10) \times T$ by its spectrum in the class of compact Kaehler submanifolds in a complex projective space (Proposition 4).

Next, by applying Ros' method, we prove that $CP^n(1/3)$ and compact irreducible Hermitian symmetric spaces of rank 3 in $CP^{n+p}(1)$ are all of order $\{1, 2, 3\}$ (Proposition 5), where $CP^m(c)$ denotes an m-dimensional complex projective space