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On the growth of meromorphic solutions of some higher order differential equations

Dedicated to Professor Y. Kusunoki on his 60th birthday

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1. Introduction.

About fifty years ago, Yosida ([**19**]) generalized a Malmquist's theorem ([8]) with the aid of the Nevanlinna theory of meromorphic functions.

THEOREM OF YOSIDA. *If the differential equation*

(1) $(w')^m = R(z, w)$, *R rational in z, w, and m a positive integer,*

possesses a transcendental meromorphic solution $w=w(z)$ in the complex plane, then $R(z, w)$ must be a polynomial in w of degree at most $2m$. Further, if $w(z)$ has only a finite number of poles, the degree is at most m.

It is well-known that this is the starting-point of applying the Nevanlinna theory to the ordinary differential equation in the complex plane. Thereafter up to the present, there are many researches in this field (e.g. see the references in [**2**], [**18**]). Among them, there are many generalizations of this theorem ([**1**], [**3**], [**6**], [**7**], [**12**], [**13**], [**14**], [**16**]).

In this paper we shall consider a general differential equation and some higher order differential equations. We denote by $\mathcal{M}$ the set of meromorphic functions in the complex plane and $\mathcal{L}$ the set of $E\subset[0, \infty)$ for which $\operatorname{meas} E<\infty$. Further, the term "meromorphic" will mean meromorphic in the complex plane.

Let P be a polynomial of $w, w', \cdots, w^{(n)}$ $(n\geqq 1)$ with coefficients in $\mathcal{M}$:

$$P=P(z, w, w', \cdots, w^{(n)})=\sum_{\lambda\in I} c_\lambda(z) w^{i_0}(w')^{i_1}\cdots(w^{(n)})^{i_n},$$

where $c_\lambda\in\mathcal{M}$ and I is a finite set of multi-indices $\lambda=(i_0, i_1, \cdots, i_n)$ for which $c_\lambda\neq 0$ and $i_0, i_1, \cdots, i_n$ are non-negative integers, and let $A(z, w)$, $B(z, w)$ be polynomials in w with coefficients in $\mathcal{M}$ and mutually prime in $\mathcal{M}$:

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