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On intuitionistic many-valued logics

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Introduction.

G. Gentzen introduced the notion of sequent, which consists of the antecedent and of the succedent, each of which in turn is a sequence of finite formulas, and utilizing that notion he formulated the formal system LK for the classical logic. Then by restricting sequents to ones whose succedents are sequences of at most one formula, he obtained from LK the formal system LJ for the intuitionistic logic. Later, Takahashi in [3], and Rousseau in [1] independently, extended the notion of sequent to that of matrix, which consists of the 1st row, the 2nd row, $\cdots$, and of the M-th row, each of which in turn is a sequence of finite formulas, where M is a natural number greater than 1, and then utilizing that notion they formulated the formal system M-LK for each M-valued logic.

What is obtained from the system M-LK, when we restrict matrices to ones whose M-th rows or more rows are sequences of at most one formula? This paper is one answer to this problem.

Let U be a subset of the non-empty finite set T of truth-values. We take a formal system for a many-valued logic having T as the set of truth-values, and then restrict every inference rule by which a connective is introduced in some μ-th row where $\mu \notin U$ so that the ν-th rows where $\nu \notin U$ of the conclusion consist of one formula in all. We call by an *intuitionistic many-valued logic* what is represented by the above-obtained system. If $U=T$, then the intuitionistic many-valued logic is of course identical with the usual many-valued logic (cf. 3.43); if $T=\{\boldsymbol{t}, \boldsymbol{f}\}$ and $U=\{\boldsymbol{f}\}$, then the logic is identical with the intuitionistic logic as is expected (cf. 3.11). Though somewhat artificial, the intuitionistic many-valued logic can also be characterized semantically (cf. Theorem 1). If either $U=T$ or U contains at most one element, then the system enjoys the cut-elimination property (cf. Theorem 4). Moreover, if U contains one and only one element, then the logic enjoys the disjunction property (cf. Theorem 5). On the contrary, if U contains at least two elements (and if sufficiently many connectives are involved), then surprisingly the intuitionistic many-valued logic coincides with the

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