

Hyperbolic links with Brunnian properties

To the memory of Professor Shichirô Oka

By Taizo KANENOBU

(Received Nov. 12, 1984)

1. Introduction.

In his paper of 1892 [3], Hermann Brunn constructed a link with what is called the Brunnian property, that is, the link itself is nontrivial, yet every proper sublink is trivial, see [21, p. 67]. His link is the origin of the links in this paper.

A k -link L in S^{k+2} is *splittable* if there exists a $(k+2)$ -disk B^{k+2} in S^{k+2} satisfying $L \cap B^{k+2} \neq \emptyset$, $L \cap \partial B^{k+2} = \emptyset$, and $L \cap (S^{k+2} - B^{k+2}) \neq \emptyset$. Generalizing the Brunnian property, H. Debrunner considered the splitting problem of a link. Given a k -link $L = L_1 \cup L_2 \cup \dots \cup L_n$ in S^{k+2} , let \mathfrak{A} be a family of those subsets S of $I_n = \{1, 2, \dots, n\}$ for which the sublink $L_S = \bigcup_{i \in S} L_i$ of L does not split. Then we call L has the *Brunnian property of type* \mathfrak{A} . For convenience, we assume that $\emptyset, \{i\} \notin \mathfrak{A}$ for all $i \in I_n$. For this family of subsets \mathfrak{A} , the following condition must be satisfied:

(*) If $S, T \in \mathfrak{A}$ and $S \cap T \neq \emptyset$, then $S \cup T \in \mathfrak{A}$.

Conversely, for any family \mathfrak{A} of subsets of I_n with (*), H. Debrunner [4] constructed a k -link with n components for each $k \geq 2$ and $n \geq 2$ having the Brunnian property of type \mathfrak{A} . Furthermore, the author [8] constructed such an example for each $k \geq 1$ and $n \geq 2$, where the link is a satellite link, that is, a link with nontrivial companions. See [8] for other references of this problem.

In this paper we restrict ourselves to 1-links in S^3 . In §4, we show that for any family \mathfrak{A} of subsets of I_n with (*), there exists a link L with Brunnian property of type \mathfrak{A} such that the exterior of each sublink L_S for $S \in \mathfrak{A}$ is atoroidal (Theorem 4), that is, irreducible and boundary-irreducible and contains no non-boundary-parallel incompressible annuli and tori. Such a link is hyperbolic by Thurston's theorem, see [13]. Note that a link is nonsplittable if and only if its exterior is irreducible.

This research was partially supported by Grant-in-Aid for Scientific Research (No. 552000152819), Ministry of Education, Science and Culture.