J. Math. Soc. Japan
Vol. 38, No. 2, 1986

Extension of modifications of ample divisors on fourfolds: II

By Maria Lucia FANIA

(Received Feb. 28, 1984)
(Revised Nov. 12, 1984)

Introduction.

In [4] I looked at the following problem: Let A be an ample divisor on a connected four dimensional projective manifold X. Assume that the Kodaira dimension of X is non negative. Suppose that A is the blow up of a projective manifold A' with center R_g where R_g is a smooth curve of genus $\geqq 1$ which is contained in A'. Does there exist a four dimensional manifold X' such that A' lies on X' as a divisor and such that X is the blow up of X' with center R_g? The answer turned out to be positive.

It was hoped that the result would still hold true for the case when $g=0$, i.e., when $R_g \simeq \boldsymbol{P}^1$. I would like to express my sincere thank to the referee for providing a counterexample in the above case. I have included this counterexample later in this paper. Hence the main theorem has been modified to obtain the following:

THEOREM. *Let X be a connected four dimensional projective variety which is a local complete intersection with isolated singularities. Assume that the ω_X-dimension of the invertible sheaf ω_X is non negative. Let A be a smooth ample divisor on X. Assume that A is the blow up of a smooth projective threefold A' with center a smooth projective curve R_g of genus $g \geqq 0$ and let Y denote the exceptional divisor on A. Then*

(i) *if $g \geqq 0$ and $Y \not\simeq \boldsymbol{P}^1 \times \boldsymbol{P}^1$ there exists a four dimensional variety X' which is a local complete intersection such that A' lies in X' as a divisor, such that X is the blow up of X' along R_g,*

(ii) *if $g=0$ and $Y \simeq \boldsymbol{P}^1 \times \boldsymbol{P}^1$* (i) *is still true unless $N_{A/X,Y} = \mathcal{O}(a, 1)$ with $a \geqq 2$. In the case when $N_{A/X,Y} = \mathcal{O}(a, 1)$, $a \geqq 2$ there exists a four dimensional Cohen-Macaulay variety X' and a morphism $\phi: X \to X'$ such that:*

a) *the following diagram commutes*