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G-maps and tangential representations at G-fixed points of G-manifolds

Dedicated to Professor Minoru Nakaoka on his sixtieth birthday

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0. Introduction.

Let G be a finite group. A G-manifold and a G-map are understood to be a *smooth* G-manifold and a *continuous* G-map respectively in this paper.

Smith equivalence has recently been studied by T. Petrie and others (e.g. [11], [19], [28]). The result in [11] is roughly described as follows. Let G be a finite cyclic group with at least four distinct primes dividing $|G|$ (the order of G). If complex G-modules V and W satisfy certain conditions concerning (i) dimension, (ii) restriction to Sylow subgroups and (iii) G-signature, then we have a rational homotopy sphere $f: \Sigma \to S(\boldsymbol{R} \oplus V \oplus U)$ with G-action such that $\Sigma^G = \{p, q\}$, $T_p\Sigma \cong V \oplus U$ and $T_q\Sigma \cong W \oplus U$ as real G-modules. Here $\boldsymbol{R}$ is the 1-dimensional real G-module with trivial action and U a complex G-module.

The purpose of this paper is to consider a similar problem for an oriented G-manifold instead of $S(\boldsymbol{R} \oplus V \oplus U)$ under several hypotheses.

Let X be a closed 1-connected oriented G-manifold with a finite number of G-fixed points $x_1, \cdots, x_n$. We denote by V_i the oriented tangential G-representations at x_i respectively for $i=1, \cdots, n$. Let Λ be the field of rational numbers $\boldsymbol{Q}$ or the ring of integers $\boldsymbol{Z}$. Let $W_1, \cdots, W_n$ be oriented G-modules. Then we have the problem:

(PΛ) Give sufficient conditions on $W_1, \cdots, W_n$ for the existence of a closed 1-connected oriented G-manifold Y and a degree one G-map $f: Y \to X$ satisfying the following properties,

(0.1) (i) f induces an isomorphism $f_*: H_*(Y; \Lambda) \to H_*(X; \Lambda)$. (ii) Y has G-fixed points $y_1, \cdots, y_n$, $n = \mathrm{card}(Y^G)$, such that $f(y_i) = x_i$ and $T_{y_i}Y \cong W_i$ as oriented real G-modules for $i=1, \cdots, n$.

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