J. Math. Soc. Japan
Vol. 38, No. 2, 1986

The L^p-boundedness of pseudodifferential operators with estimates of parabolic type and product type

By Masao YAMAZAKI

(Received May 18, 1984)
(Revised Oct. 12, 1984)

§ 0. Introduction.

In this paper we consider symbols $P(x, \xi)$ on $\boldsymbol{R}^n$ whose derivatives do not necessarily converge to 0 as $|\xi|\rightarrow\infty$, and we give some sufficient conditions for the L^p-boundedness of the associated pseudodifferential operators $P(x, D)$. Some modifications of the Fourier multiplier theorem of Mikhlin type and Stein type are also obtained, together with those of the Littleweed-Paley decomposition of the space $L^p(\boldsymbol{R}^n)$. Part of the results of this paper has been announced in Yamazaki [15].

The L^p-boundedness of pseudodifferential operators on $\boldsymbol{R}^n$ with non-smooth symbols has been studied by many authors. See Mossaheb-Okada [8], Nagase [10], Coifman-Meyer [4], Muramatu-Nagase [9] and Bourdaud [2]. They considered symbols $P(x, \xi)$ on $\boldsymbol{R}^n$ satisfying the estimate $|\partial_\xi^\alpha P(x, \xi)|\leqq C_\alpha(1+|\xi|)^{-|\alpha|}$ for every multi-index α satisfying $|\alpha|\leqq n+1$ (or $|\alpha|\leqq n+2$), and obtained the L^p-boundedness of the associated pseudodifferential operators $P(x, D)$ defined by the formula

$$P(x, D)u(x)=\int e^{ix\cdot\xi}P(x, \xi)\hat{u}(\xi)\bar{d}\xi$$

under some assumptions on the regularity of the symbol $P(x, \xi)$ with respect to x. Here $\bar{d}\xi$ denotes $(2\pi)^{-n}d\xi$, and $\hat{u}(\xi)$ denotes the Fourier transform of $u(x)$. Here and hereafter we assume $1<p<\infty$ and denote $L^p=L^p(\boldsymbol{R}^n)$, and the integrals are done over $\boldsymbol{R}^n$ unless otherwise specified.

On the other hand, Stein [11] proved the L^p-boundedness of the Fourier multiplier $m(\xi)$ satisfying the estimates $|\xi^\alpha\partial_\xi^\alpha m(\xi)|\leqq C$ for all $\alpha\in\boldsymbol{N}^n$ such that $\alpha_l=0$ or 1 for every $l=1, 2, \cdots, n$. Here the space $\boldsymbol{R}^n$ is regarded as the direct product of n copies of $\boldsymbol{R}$.

Fefferman [6] and Fefferman-Stein [7] regarded $\boldsymbol{R}^n$ as $\boldsymbol{R}^{n-l}\times\boldsymbol{R}^l$, and obtained several boundedness properties of the singular integrals with kernels $K(y, z)$ $(y\in\boldsymbol{R}^{n-l}, z\in\boldsymbol{R}^l)$ satisfying the estimate $|K(y, z)|\leqq C|y|^{-n+l}|z|^{-l}$ under some hypotheses.