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Rings of automorphic forms which are not Cohen-Macaulay, I

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By Noether's normalization theorem, a noetherian graded algebra R has a polynomial subring S generated by homogeneous elements such that R is finite over S. It is known (see, for instance Stanley [24], § 3) that R is Cohen-Macaulay (C.-M., for short) if and only if R is free over any (equivalently some) such S. Thus it is meaningful to ask which of the graded rings of automorphic forms are C.-M. This is a problem posed by Eichler [4], [5]. Igusa [16] determined the structure of the graded rings of Siegel modular forms of degree two for groups containing $\Gamma_2(2)$, and Resnikoff and Tai [20], [26] determined the structure of the graded rings of automorphic forms on the complex 2-ball for some arithmetic group. These rings turn out to be C.-M. However Freitag [6] showed that the ring of Hilbert modular forms of degree $\geqq 3$ is not C.-M., while in our previous paper [27], we proved that the ring of Hilbert modular forms of even weight and of degree two is C.-M. In this paper we show that the ring of automorphic forms fails to be C.-M. for a large class of neat arithmetic groups as well as for the Siegel modular group $\Gamma_g = Sp_{2g}(\boldsymbol{Z})$, $g \geqq 4$.

Samuel [22] stated "All the examples of U.F.D.'s I know are C.-M. Is it true in general?" (see Lipman [19] for the history of this question). In the case of characteristic zero, Freitag and Kiehl [9] gave a negative answer to this question of Samuel by constructing analytic local rings which are U.F.D.'s of dimension 60 and depth 3, hence not C.-M. As far as we know these are the only previously known examples. As Freitag [7], [8] has shown, the ring of Siegel modular forms for Γ_g ($g \geqq 3$) is U.F.D. Hence our result shows that they furnish negative examples to Samuel's question in arbitrary high dimension.

To prove our assertion it is enough to prove that the Baily-Borel compactification of the corresponding quotient space is not a C.-M. scheme, where a C.-M. scheme is a scheme whose local rings are all C.-M. This gives some generalization of the result of Igusa [17], where he shows that the Baily-Borel compactification does not admit a finite nonsingular covering under some condition on the bounded symmetric domain and the arithmetic group.

This work was done while the author was staying at Harvard University.