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Functional calculus for certain Banach function algebras

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In this paper we study the symbolic calculus for a Banach function algebra with certain conditions. First we define a class of Banach function algebras which contains the class of all function algebras and the class of all ultraseparating Banach function algebras. Our purpose is to prove the theorem asserting that if A is a non-trivial Banach function algebra in the class, then only analytic functions can operate on A. It is a generalization of theorems of de Leeuw and Katznelson [**6**], Bernard [**2**] and Bernard and Dufresnoy [**3**].

1. Introduction.

Let A be a Banach function algebra on a compact Hausdorff space X, that is, a point separating unital subalgebra of $C(X)$ (the algebra of all complex valued continuous functions on X) with the Banach algebraic norm $N(\cdot)$. We say that A is a function algebra if $N(\cdot)$ is the supremum norm $\|\cdot\|_\infty$. Suppose that h is a complex valued continuous function on an open subset D of the complex plane. We say that h operates by composition on A if $h\circ f$ is in A whenever $f\in A$ has the range contained in D.

de Leeuw-Katznelson [**6**] proved that if A is a non-trivial function algebra ($=$non $C(X)$), the nany non-analytic function does not operate by composition on A. But it is not the case for Banach function algebras as many examples show. For example the algebra

$$A(\Gamma)=\{f\in C(\Gamma):\ \sum|\hat{f}(n)|<\infty\}$$

of all continuous functions on the unit circle Γ with absolutely convergent Fourier series with the norm

$$N(f)=\sum|\hat{f}(n)|$$

for f in $A(\Gamma)$, where $\hat{f}(n)$ denotes n-th Fourier coefficients, is conjugate closed non-trivial Banach function algebra. Bernard [**2**] defined ultraseparability for Banach function algebras and showed that $\bar{z}$ does not operate by composition on a non-trivial ultraseparating Banach function algebra. Bernard-Dufresnoy [**3**]