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On Z_p-extensions of real quadratic fields

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§ 0. Introduction.

Let k be a finite totally real extension of $\boldsymbol{Q}$, and p an odd prime number. Concerning the Greenberg's conjecture (cf. [2]) which states that Iwasawa invariants $\mu_p(k)$ and $\lambda_p(k)$ both vanish, we have obtained some results in the previous paper [1]. The purpose of this paper is to extend the results in our previous work.

For a finite algebraic number field K, we denote by h_K, C_K, and E_K the class number of K, the ideal class group of K, and the unit group of K, respectively. We denote also by $|X|$ the cardinality of a finite set X.

In the following, we assume that k is a real quadratic field and ε denotes the fundamental unit of k. Let p be an odd prime number which splits in $k/\boldsymbol{Q}$, and $\mathfrak{P}$ a prime of k lying above p. Take $\alpha \in k$ such that $\mathfrak{P}^{h_k}=(\alpha)$. We define n_1 (resp. n_2) to be the maximal integer such that $\alpha^{p-1}\equiv 1 \pmod{p^{n_1}\boldsymbol{Z}_p}$ (resp. $\varepsilon^{p-1}\equiv 1 \pmod{p^{n_2}\boldsymbol{Z}_p}$). Note that n_1 is uniquely determined under the condition $n_1 \leqq n_2$. For the cyclotomic $\boldsymbol{Z}_p$-extension

$$k=k_0 \subset k_1 \subset k_2 \subset \cdots \subset k_n \subset \cdots \subset k_\infty,$$

let A_n be the p-primary part of the ideal class group of k_n, B_n the subgroup of A_n consisting of ideal classes which are invariant under the action of $\mathrm{Gal}(k_n/k)$, and D_n the subgroup of A_n consisting of ideal classes which contain a product of ideals lying over p. Let E_n be the unit group of k_n. For $m \geqq n \geqq 0$, $N_{m,n}$ denote the norm maps. We fix a topological generator σ of $G(k_\infty/k)$. Let ζ_p be a primitive p-th root of unity, and A_0^* the p-primary part of the ideal class group of $k(\zeta_p)$. Our main theorems are

THEOREM 1. *Let k be a real quadratic field and p an odd prime number which splits in $k/\boldsymbol{Q}$. Assume that*

(1) $n_1=1$, *and*

(2) $A_0=D_0$.

Then, for $n \geqq n_2-1$, we have $|A_n|=|D_n|=|D_0|\cdot p^{n_2-1}$.

Concerning the Iwasawa invariants $\mu_p(k)$, $\lambda_p(k)$ and $\nu_p(k)$, we obtain the next corollary.