

## On indefinite modular forms of weight one

By Toyokazu HIRAMATSU, Noburo ISHII  
and Yoshio MIMURA

(Received Sept. 5, 1984)

### Introduction.

As shown in our previous papers ([3], [4]), there are deep relations between the class fields over imaginary quadratic fields and cusp forms of weight one of "neben typus" in Hecke's sense. In this paper we study a similar problem for class fields over real quadratic fields which satisfy a condition due to Shintani ([13]). The paper consists of five sections. In Section 1 we recall the definition of Hecke's indefinite modular forms of weight one which are associated to real quadratic fields ([1], [2], [10]). In Section 2 we summarize certain results of Shintani for the real quadratic problem which is transferable to the imaginary quadratic situation ([13]). In Section 3 we apply the result of Shintani to our problem and obtain the two representations for some dihedral cusp forms of weight one by positive definite theta series and indefinite theta series. Kac and Peterson in [7] gave many examples of new identities for cusp forms of weight one which arise from the Dedekind eta function. In Section 4 we shall reconstruct these examples from our point of view, by using the results of Section 3. In the final section we establish the higher reciprocity law for a defining equation of ray class fields over some real quadratic fields.

The authors would like to express their sincere thanks to Professor N. Iwahori for informing them of the work of Kac and Peterson ([7]). Our work has been particularly inspired by this work.

### 1. Hecke's indefinite modular forms of weight one.

In this section we shall review the definition and basic properties of the indefinite modular forms which were introduced by Hecke ([1], [2]).

Let  $F$  be a real quadratic field with discriminant  $D$ , and  $\mathfrak{o}_F$  the ring of all integers in  $F$ . Let  $Q$  be a natural number and denote by  $u_0$  the group of totally positive units  $\varepsilon$  of  $\mathfrak{o}_F$  such that  $\varepsilon \equiv 1 \pmod{Q\sqrt{D}}$ . Let  $\mathfrak{a}$  be an integral ideal of  $\mathfrak{o}_F$ , and put

$$|N(\mathfrak{a})| = A.$$